CHAPTER 4f.
ROOTS OF EQUATIONS

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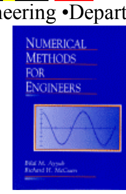

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Spring 2001

ENCE 203 - Computation Methods in Civil Engineering II

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Multiple Roots

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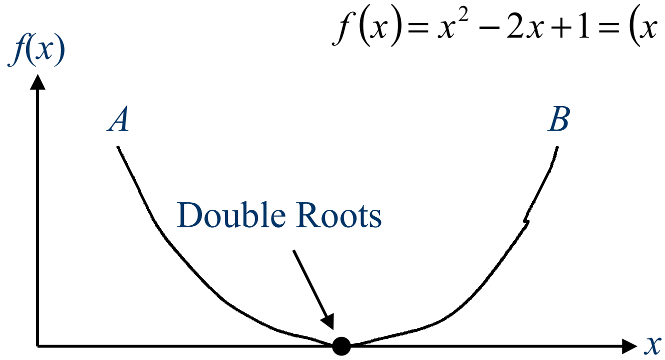
- The following figure shows a case of multiple roots, where the function *f*(*x*) is tangent to the x axis.
- This case corresponds to having two roots of the same value and sign

Multiple Roots

- Double Roots

$$f(x) = x^2 - 2x + 1 = (x-1)^2$$



Multiple Roots

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- In general, there can be triple, quadruple, …., or multiple roots for a function *f*(*x*).
- It can be shown that even roots result in tangent *f*(*x*) to the x axis, whereas odd multiple roots result in a function *f*(*x*) that crosses the *x* axis with inflection point at the root.

Multiple Roots

- That is, a point where the function changes curvature.
- Example:
  - The following function has triple roots:

$$f(x) = x^4 - 8x^3 + 18x^2 - 16x + 5$$
$$= (x-1)(x-1)(x-1)(x-5)$$

Multiple Roots

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$$f(x) = x^4 - 8x^3 + 18x^2 - 16x + 5$$
$$= (x-1)(x-1)(x-1)(x-5)$$

- This function has triple roots (x = 1) and one root (x = 5) as shown in the following figure.
- The figure shows the inflection point at x = 1 that result in $f(x)$ = 0

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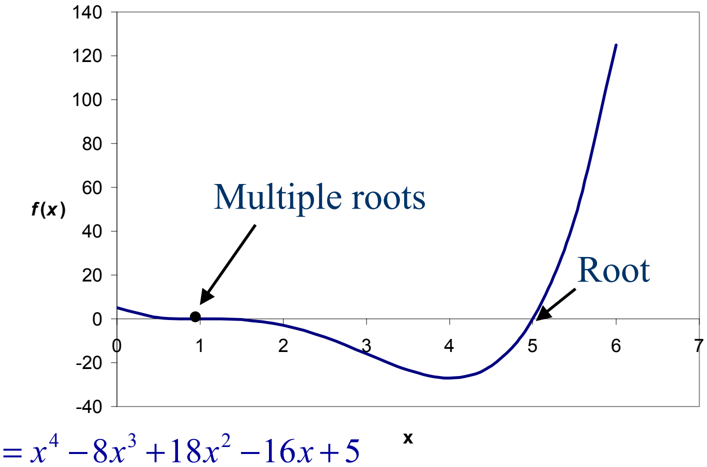
Multiple Roots



$$f(x) = x^4 - 8x^3 + 18x^2 - 16x + 5$$

Multiple Roots

- Problems with Multiple Roots
  - Multiple roots pose difficulties for the methods discussed so far.
  - The bisection method has difficulties with multiple roots because the function does not change sign at even multiple roots.
  - The Newton-Raphson and secant methods have difficulties because the derivative at a multiple root is zero.

Multiple Roots

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- Problems with Multiple Roots
  - Since $f(x)$ reaches zero at a faster rate than $f'(x)$ as x approaches the multiple root, it is impossible to check for the condition $f(x) = 0$ and terminate the computations before $f'(x) = 0$.

Systems of Nonlinear Equations

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- The methods introduced so far for finding the roots of a function deal with single-variable equations of the type $f(x) = 0$.
- Some engineering problems have two or more variables for which the roots are needed.

Systems of Nonlinear Equations

- Two-Variable Problem
  - For two-variable problems, the function has the form

$$f_i(x, y) = 0$$

where the subscript *i* denotes the equation number, and both *x* and *y* are independent variables

Systems of Nonlinear Equations

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- Example

The following is an example of a two-variable system of equations:

$$x^3 - 3x^2 + xy = 0$$

$$4x^2 - 4xy^2 + 3y^2 = 0$$

The methods used for single-variable cannot be applied directly to find the values of *x* and *y*.

Systems of Nonlinear Equations

- Example (cont'd)

$$x^3 - 3x^2 + xy = 0$$

$$4x^2 - 4xy^2 + 3y^2 = 0$$

The equations can be solved for *x* and *y* as follows:

$$x = \left(3x^2 - xy\right)^{1/3}$$

$$y = \sqrt{\left(\frac{4x^2 + 3y^2}{4x}\right)}$$

Systems of Nonlinear Equations

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- Example (cont'd)

Using initial guesses: *x* = 3 and *y* =3, we have

$$x = \left[3(3)^2 - (3)(3)\right]^{1/3} = 2.621$$

$$\boxed{\begin{aligned} x &= \left(3x^2 - xy\right)^{1/3} \\ y &= \sqrt{\left(\frac{4x^2 + 3y^2}{4x}\right)} \end{aligned}}$$

Using *x* = 2.621 and *y* = 3 in the second equation, gives

$$y = \sqrt{\frac{4(2.621)^2 + 2(3)^2}{4(2.621)}} = 2.280$$

Systems of Nonlinear Equations

$$x = \left(3x^2 - xy\right)^{1/3}$$
$$y = \sqrt{\left(\frac{4x^2 + 3y^2}{4x}\right)}$$

- Example (cont'd)

  For the second iteration:

  - Use x = 2.621 and y = 2.280, therefore

$$x = \left[3(2.621)^2 - (2.621)(2.280)\right]^{1/3} = 2.446$$

  - Use x = 2.446 and y = 2.280, therefore

$$y = \sqrt{\frac{4(2.446)^2 + 3(2.280)^2}{4(2.446)}} = 2.010$$

Systems of Nonlinear Equations

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- Example (cont'd)

  The solution will eventually converge to the following roots (see Table 4-9 of Textbook):

  x = 2.16

  y = 1.82