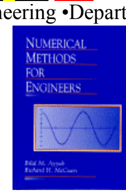
CHAPTER 2b.
MATRICES



by

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Types of Matrices

- Unit or Identity Matrix
  - A *unit* (*identity*) *matrix* is a diagonal matrix with all the elements in the principal diagonal equal to one.
  - The identity or unit matrix, designated by *I* is worthy of special consideration.
  - For any arbitrary matrix ***A***, the following relationships hold true:

$$\boldsymbol{AI} = \boldsymbol{A} \quad \text{and} \quad \boldsymbol{IA} = \boldsymbol{A}$$

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Types of Matrices

- Unit or Identity Matrix
  - Examples:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{If the matrix } A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 2 & 5 & 6 \end{bmatrix} \quad \text{then} \quad AI = IA = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 2 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 2 & 5 & 6 \end{bmatrix}$$

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Types of Matrices

- Null or Zero Matrix
  - A *null* (*zero*) *matrix* is any matrix in which all the elements have zero values. It is usually denoted as ***0***.
  - Examples:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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Types of Matrices

- Symmetric Matrix
  - A symmetric matrix is a square matrix in which $a_{ij} = a_{ji}$.
  - Examples:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & 7 & 9 \\ 3 & 4 & 2 & 10 \\ 7 & 2 & 7 & 8 \\ 9 & 10 & 8 & 11 \end{bmatrix}$$

$a_{13} = a_{31}$ $\qquad$ $a_{23} = a_{32}$

Types of Matrices

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- Skew Symmetric
  - A *skew-symmetric matrix* is square matrix with all values on the principal diagonal equal to zero and with off-diagonal values given such that $a_{ij} = -a_{ji}$.
  - Examples:

$$\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 5 & -9 \\ -2 & 0 & -4 & -6 \\ -5 & 4 & 0 & 12 \\ 9 & 6 & -12 & 0 \end{bmatrix}$$

Types of Matrices

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- Transposed Matrix
  - Given a matrix ***A***, the *transpose* of ***A***, denoted by A^T and read *A*-transpose, is obtained by changing all the rows of ***A*** into the columns of A^T while preserving the order.
  - Hence, the first row of ***A*** becomes the first column of A^T, while the second row of ***A*** becomes the second column of A^T, and the last row of ***A*** becomes the last column of A^T.

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- Transposed Matrix
  - In terms of the elements, $a_{ij}^T = a_{ji}$
  - If matrix ***A*** has *r* rows and *c* columns, then A^T will have *c* rows and *r* columns
  - Note that

$$\left(A^T\right)^T = A$$

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■ Examples Transposed Matrix

– Thus if

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad \text{then } A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

– and if

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}, \quad \text{then } \; A^T = \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}$$

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Matrix Operations

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■ The primary arithmetic operations are
- – Addition
- – Subtraction
- – Multiplication, and
- – Division

■ Matrix algebra has operations called addition, subtraction, and multiplication.

■ No division in matrix algebra, instead there is <u>matrix inversion</u>

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Matrix Operations

■ Matrix Equality

– The simplest relationship between two matrices is equality.

– Intuitively, one feels that two matrices should be equal if their corresponding elements are equal.

– This the case provided that the two matrices are of the same order (size).

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Matrix Operations

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■ Matrix Equality

– Two matrices **A** = $[a_{ij}]_{r\times c}$ and **B** = $[b_{ij}]_{r\times c}$ are equal if they have the same order and if $a_{ij} = a_{ij}$ for all i and j.

– Thus, the equality

$$\begin{bmatrix} 5x+2y \\ x-3y \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

implies that

$$5x+2y=7 \quad \text{and} \quad x-3y=1$$

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Matrix Operations

- Matrix Addition
  - If $\boldsymbol{A} = [a_{ij}]_{r\times c}$ and $\boldsymbol{B} = [b_{ij}]_{r\times c}$ are both of order (size) $r \times c$, then $\boldsymbol{A} + \boldsymbol{B}$ is a $r \times c$ matrix $[c_{ij}]_{r\times c}$ where

$$c_{ij} = a_{ij} + b_{ij}$$

Matrix Operations

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- Matrix Addition
  - Examples:

$$\begin{bmatrix} 5 & 1 \\ 7 & 3 \\ -2 & -1 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ 2 & -1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 5+(-6) & 1+3 \\ 7+2 & 3+(-1) \\ (-2)+4 & (-1)+1 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 9 & 2 \\ 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} t^2 & 5 \\ 3t & 0 \end{bmatrix} + \begin{bmatrix} 1 & -6 \\ t & -t \end{bmatrix} = \begin{bmatrix} t^2+1 & -1 \\ 4t & -t \end{bmatrix}$$

Matrix Operations

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■ Matrix Addition

– Examples:

$$\text{If } A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 9 & 10 & 11 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2 & 3 \\ 7 & 8 & 4 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\text{Then, } C = A + B = \begin{bmatrix} 2 & 6 & 9 \\ 8 & 11 & 9 \\ 11 & 11 & 12 \end{bmatrix}$$

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■ Matrix Addition

– Examples:

- The following matrices ***A*** and ***B*** cannot be added since they are not of the same order.

$$A = \begin{bmatrix} 5 & 0 \\ -1 & 0 \\ 2 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -6 & 2 \\ 1 & 1 \end{bmatrix}$$

- The equality ***C*** = ***A*** + ***B*** is not defined.

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Matrix Operations

- Matrix Subtraction
  - If ***A*** = $[a_{ij}]_{r\times c}$ and ***B*** = $[b_{ij}]_{r\times c}$ are both of order (size) $r \times c$, then ***A*** - ***B*** is a $r \times c$ matrix $[c_{ij}]_{r\times c}$ where

$$c_{ij} = a_{ij} - b_{ij}$$

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- Matrix Subtraction
  - Example:

$$\text{If } A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 9 & 10 & 11 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2 & -3 \\ 7 & -8 & 4 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\text{Then, } C = A - B = \begin{bmatrix} 2-0 & 4-2 & 6-(-3) \\ 1-7 & 3-(-8) & 5-4 \\ 9-2 & 10-1 & 11-1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 9 \\ -6 & 11 & 1 \\ 7 & 9 & 10 \end{bmatrix}$$

Matrix Operations

■ Commutative and Associative Laws

– If ***A***, ***B***, and ***C*** represent matrices of the same order, then

1) $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
2) $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$
3) $\mathbf{A} + \mathbf{0} = \mathbf{A}$

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Matrix Operations

■ Commutative Law

– Matrix addition is not directional (commutative), that is

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathrm{A}$$

– Matrix subtraction is directional (noncommutative), that is

$$\mathbf{A} - \mathbf{B} \neq \mathbf{B} - \mathbf{A}$$

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Matrix Operations

- Matrix Multiplication
  - Two matrices are *not* multiplied together elementwise.
  - It is not possible to multiply matrices of the same order while it is possible to multiply certain matrices of different orders.
  - If ***A*** and ***B*** are two matrices for which multiplication is defined, it is generally not the case that ***AB*** = ***BA***.

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Matrix Operations

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- Matrix Multiplication
  - General Rules
    - Using ***A***, ***B***, and ***C*** to denote three matrices for the matrix product ***C*** = ***AB***, the following are the rules for matrix multiplication:
      1. The number of columns in the first matrix ***A*** must equal the number of rows in the second matrix ***B***.
      2. The number of rows in the product matrix ***C*** equals the number of rows in the first matrix ***A***.
      3. The number of columns in the product matrix ***C*** equals the number of columns in the second matrix ***B***.

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Matrix Operations

- Matrix Multiplication
  - General Rules
    4. The element of matrix ***C*** in row *i* and column *j* (c_{ij}) is equal to the sum of the products $a_{ik}\, b_{kj}$:

$$c_{ij} = \sum_{k=1}^{m} a_{ik} b_{kj}$$

where *m* is the number of columns in ***A***, which is also the number of rows in ***B***.

Matrix Operations

- Matrix Multiplication
  - General Rules
    5. Matrix multiplication is not commutative, that is,

$$\boldsymbol{AB} \neq \boldsymbol{BA}$$

    6. Matrix multiplication is associative, that is,

$$(\boldsymbol{AB})\,\boldsymbol{C} = \boldsymbol{A}\,(\boldsymbol{BC})$$

Matrix Operations

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- Matrix Multiplication
  - Multiplication Terms
    - Premultiplication of ***B*** by ***A*** means ***AB***.
    - Premultiplication of ***A*** by ***B*** means ***BA***.
    - Post multiplication of ***A*** by ***B*** means ***AB***.
    - Post multiplication of ***B*** by ***A*** means ***BA***.

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- Matrix Multiplication
  - Rule 1:

    The product of two matrices ***AB*** is defined if the number of columns of ***A*** equals the number of rows of ***B***

    Thus, if ***A*** and ***B*** are given by

$$A = \begin{bmatrix} 6 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 3 & 2 & -2 & 1 \\ 4 & 1 & 1 & 0 \end{bmatrix}$$

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- Matrix Multiplication
  - Rule 1(cont'd):

    then the product ***AB*** is defined since ***A*** has three columns and ***B*** has three rows. The product ***BA***, however is not defined since ***B*** has four columns while ***A*** has only two rows.

    When the product is written as ***AB***, ***A*** is said to *premultiply* ***B*** while ***B*** is said to *postmultiply* ***A***.

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- Matrix Multiplication
  - Rule 2:

    If the product ***AB*** is defined, then the resultant matrix will have the same number of rows as ***A*** and the same number of columns as ***B.***

    Thus, if ***A*** and ***B*** are given by

$$A = \begin{bmatrix} 6 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 3 & 2 & -2 & 1 \\ 4 & 1 & 1 & 0 \end{bmatrix}$$

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- Matrix Multiplication
  - Rule 2(cont'd):

    then, the product ***C*** = ***AB*** will have two rows and four columns since ***A*** has two rows and ***B*** has four columns.

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- Matrix Multiplication
  - Easy Method for Rules 1 and 2
    - Write the orders of the matrices on paper in the sequence in which the multiplication is to be carried out, that is, if ***AB*** is to be found where ***A*** has order $(r_A \times c_A)$ and ***B*** has order $(r_B \times c_B)$, write

$$(r_A \times c_A)\ (r_B \times c_B)$$

Matrix Operations

- Matrix Multiplication
  - Easy Method for Rules 1 and 2 (cont'd)
    - If the two adjacent numbers (indicated by the arrows) c_A and r_B are equal, then the multiplication is defined.
    - The order of the product matrix ***C*** = ***AB*** is obtained by canceling the adjacent numbers and using the two remaining numbers, that is, the order of ***C*** is $r_A \times c_B$

$$(r_A \times c_A)(r_B \times c_B)$$

order of ***C***

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Matrix Operations

- Matrix Multiplication
  - Easy Method for Rules 1 and 2 (cont'd
    - Examples:

$$A = \begin{bmatrix} 6 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 3 & 2 & -2 & 1 \\ 4 & 1 & 1 & 0 \end{bmatrix}$$

Order: 2×3 Order 3×4

$$(2 \times 3)\ (3 \times 4)$$

The matrix product ***C*** = ***AB*** is defined.
The order of ***AB*** is 2×4

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Matrix Operations

■ Matrix Multiplication

– Easy Method for Rules 1 and 2 (cont'd

• Examples:

$$B = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 3 & 2 & -2 & 1 \\ 4 & 1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 6 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

Order 3×4 Order: 2×3

$$(3 \times 4)\ (2 \times 3)$$

The matrix product $\boldsymbol{C} = \boldsymbol{BA}$ is not defined.

Matrix Operations

■ Matrix Multiplication

– Rule 3:

If the matrix product ***AB*** is defined, where ***C*** is denoted by $[c_{ij}]$, then the element c_{ij} is obtained by multiplying the elements in the i^{th} row of ***A*** by the corresponding elements in the j^{th} column of ***B*** and adding.

Thus, if ***A*** has order $r_A \times c_A$, ***B*** has order $r_B \times c_B$, $c_A = r_B$, and

Matrix Operations

– Rule 3 (cont'd):

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1c_A} \\ a_{21} & a_{22} & \dots & a_{2c_A} \\ \vdots & \vdots & \vdots & \vdots \\ a_{r_A1} & a_{r_A2} & \dots & a_{r_Ac_A} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1c_B} \\ b_{21} & b_{22} & \dots & b_{2c_B} \\ \vdots & \vdots & \vdots & \vdots \\ b_{r_B1} & b_{r_B2} & \cdots & b_{r_Bc_B} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{21} & \dots & c_{1c_C} \\ c_{21} & c_{22} & \dots & c_{2c_C} \\ \vdots & \vdots & \vdots & \vdots \\ c_{r_C1} & c_{r_C2} & \cdots & c_{r_Cc_C} \end{bmatrix}$$

then, c_{11} is obtained by multiplying the elements in the first row of ***A*** by the corresponding elements in the first column of ***B*** and adding; hence,

$$c_{11} = a_{11}b_{11} + a_{12}b_{21} + \cdots + a_{1c_A}b_{r_B1}$$

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– Rule 3 (cont'd):

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1c_A} \\ a_{21} & a_{22} & \dots & a_{2c_A} \\ \vdots & \vdots & \vdots & \vdots \\ a_{r_A1} & a_{r_A2} & \dots & a_{r_Ac_A} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1c_B} \\ b_{21} & b_{22} & \dots & b_{2c_B} \\ \vdots & \vdots & \vdots & \vdots \\ b_{r_B1} & b_{r_B2} & \cdots & b_{r_Bc_B} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{21} & \dots & c_{1c_C} \\ c_{21} & c_{22} & \dots & c_{2c_C} \\ \vdots & \vdots & \vdots & \vdots \\ c_{r_C1} & c_{r_C2} & \cdots & c_{r_Cc_C} \end{bmatrix}$$

the element c_{12} is obtained by multiplying the elements in the first row of ***A*** by the corresponding elements in the second column of ***B*** and adding; hence,

$$c_{12} = a_{11}b_{12} + a_{12}b_{22} + \cdots + a_{1c_A}b_{r_B2}$$

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– Rule 3 (cont'd):

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1c_A} \\ a_{21} & a_{22} & \cdots & a_{2c_A} \\ \vdots & \vdots & \vdots & \vdots \\ a_{r_A1} & a_{r_A2} & \cdots & a_{r_Ac_A} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1c_B} \\ b_{21} & b_{22} & \cdots & b_{2c_B} \\ \vdots & \vdots & \vdots & \vdots \\ b_{r_B1} & b_{r_B2} & \cdots & b_{r_Bc_B} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{21} & \cdots & c_{1c_C} \\ c_{21} & c_{22} & \cdots & c_{2c_C} \\ \vdots & \vdots & \vdots & \vdots \\ c_{r_C1} & c_{r_C2} & \cdots & c_{r_Cc_C} \end{bmatrix}$$

the element $c_{r_Cc_C}$ is obtained by multiplying the elements in the first row of ***A*** by the corresponding elements in the second column of ***B*** and adding; hence,

$$c_{r_Cc_C} = a_{r_A1}b_{1c_B} + a_{r_A2}b_{2c_B} + \cdots + a_{r_Ac_A}b_{r_Bc_B}$$

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Matrix Operations

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■ Example: Matrix Multiplication

Find ***AB*** and ***BA*** if

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -7 & -8 \\ 9 & 0 \\ 0 & -11 \end{bmatrix}$$

order (2×3) order (3×2)

$$(2 \times 3)\ (3 \times 2)$$

The matrix product ***AB*** is defined with an order of 2×2

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Matrix Operations

■ Example (cont'd): Matrix Multiplication

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} -7 & -8 \\ 9 & 10 \\ 0 & -11 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-7)+2(9)+3(0) & 1(-8)+2(10)+3(-11) \\ 4(-7)+5(9)+6(0) & 4(-8)+5(10)+6(-11) \end{bmatrix}$$

$$= \begin{bmatrix} -7+18+0 & -8+20-33 \\ -28+45+0 & -32+50-66 \end{bmatrix} = \begin{bmatrix} 11 & -21 \\ 17 & -48 \end{bmatrix}$$

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■ Example (cont'd): Matrix Multiplication

$$B = \begin{bmatrix} -7 & -8 \\ 9 & 0 \\ 0 & -11 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

order (3×2) order (2×3)

$$(3 \times 2)\ (2 \times 3)$$

The matrix product ***BA*** is defined with an order of 3×3

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Matrix Operations

■ Example (cont'd): Matrix Multiplication

$$BA = \begin{bmatrix} -7 & -8 \\ 9 & 10 \\ 0 & -11 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} (-7)1+(-8)4 & (-7)2+(-8)5 & (-7)3+(-8)6 \\ 9(1)+10(4) & (9)2+(10)5 & 9(3)+10(6) \\ (0)1+(-11)4 & (0)2+(-11)5 & (0)3+(-11)6 \end{bmatrix}$$

$$= \begin{bmatrix} -7-32 & -14-40 & -21-48 \\ 9+40 & 18+50 & 27+60 \\ 0-44 & 0-55 & 0-66 \end{bmatrix} = \begin{bmatrix} -39 & -54 & -69 \\ 49 & 68 & 87 \\ -44 & -55 & -66 \end{bmatrix}$$

Matrix Operations

■ Matrix Multiplication by a Scalar

– The multiplication of a matrix ***A*** by a scalar *s* has the effect of multiplying each element a_{ij} in the matrix by the scalar. The resulting elements of a matrix ***B*** can be expressed as

$$b_{ij} = sa_{ij}$$

Matrix Operations

- Matrix Multiplication by a Scalar
  - Example:

    Find ***B*** = *s**A*** if *s* = 5, and

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 9 & 10 & 11 \end{bmatrix}$$

$$B = sA = 5\begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 0 \\ 9 & 10 & 11 \end{bmatrix} = \begin{bmatrix} 5(2) & 5(4) & 5(6) \\ 5(1) & 5(3) & 5(0) \\ 5(9) & 5(10) & 5(11) \end{bmatrix} = \begin{bmatrix} 10 & 20 & 30 \\ 5 & 15 & 0 \\ 45 & 50 & 55 \end{bmatrix}$$

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