CHAPTER 2a. MATRICES

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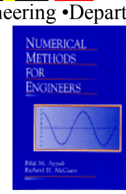

by

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ENCE 203 - Computation Methods in Civil Engineering II

Department of Civil and Environmental Engineering

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Introduction

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- Definition of a Matrix

> A *matrix* is a rectangular array of elements arranged in a horizontal rows and vertical columns.

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■ Examples:

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 4\theta_1 & \theta_2 & -\frac{3}{L}d & \frac{wL^3}{24E_1} \\ \theta_1 & 4\theta_2 & -\frac{3}{L}d & \frac{-wL^3}{24E_1} \\ \theta_1 & \theta_2 & -\frac{4}{L}d & \frac{PL^2}{6EI} \end{bmatrix}$$

$$\begin{bmatrix} \text{republican} & 37 & \text{good} & \text{not qualified} \\ \text{democrat} & 48 & \text{fair} & \text{qualified} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & 1 \\ 3 & 2 & 1 \\ 0 & 4 & 2 \end{bmatrix} \quad \begin{bmatrix} x^2 & x \\ e^x & \frac{d}{dx}\ln x \\ 5 & x+2 \end{bmatrix} \quad \begin{bmatrix} \sqrt{2} \\ \pi \\ 19.5 \end{bmatrix}$$

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■ General Form of a Matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1c} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2c} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3c} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{r1} & a_{r2} & a_{r3} & \dots & a_{rc} \end{bmatrix} \quad (1)$$

Introduction

- General Form of a Matrix
  - Each column has the same number of values *r*

    r = number of rows
  - Each row has the same number of values *c*

    c = number of columns
  - It is not necessary for the number of rows to equal the number of columns

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- General From of a Matrix
  - Notations and Convention
    - A capital letter can be used to denote the name of an array or a matrix.
    - The values contained in the matrix are called elements and are denoted as a_{ij}

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{33} \end{bmatrix} \qquad \mathrm{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{33} \end{bmatrix}$$

Introduction

■ General From of a Matrix

– Notations and Convention

- A matrix can also be abbreviated using the following notations:

$$\lfloor a_{ij} \rfloor_{r \times c} \qquad \textbf{\textit{or simply}} \qquad \lfloor a_{ij} \rfloor$$

- In this notation, a_{ij} represents the general element of the matrix and appears in the i^{th} row and j^{th} column.
- The subscript *i*, which represents the row, can have any value 1 through *r*.

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■ General From of a Matrix

– Notations and Convention

- The subscript *j*, which represents the column, can have values from 1 to *c*.
- Thus, if *i* = 2 and *j* = 3, a_{ij} becomes a_{23} and designates the element in the second row and third column.
- if *i* = 1 and *j* = 5, a_{ij} becomes a_{15} and designates the element in the first row and fifth column.

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■ Order or Size of a Matrix

– The order (size) of a matrix is given by

Order = $r \times c$

By convention, $r \times c$ is read r by c, and the row index is always given first.

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■ Order or Size of a Matrix

– Examples:

$$\begin{bmatrix} x^2 & x \\ e^x & \frac{d}{dx}\ln x \\ 5 & x+2 \end{bmatrix}$$

Size: 3×3

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 0 & -1 \end{bmatrix}$$

Size: 2×3

$$\begin{bmatrix} 4\theta_1 & \theta_2 & -\frac{3}{L}d & \frac{wL^3}{24E} \\ \theta_1 & 4\theta_2 & -\frac{3}{L}d & \frac{-wL^3}{24E} \\ \theta_1 & \theta_2 & -\frac{4}{L}d & \frac{PL^2}{6EI} \end{bmatrix}$$

Size: 3×4

$$\begin{bmatrix} \sqrt{2} \\ \pi \\ 19.5 \end{bmatrix}$$

Size: 3×1

$$\begin{bmatrix} \sqrt{2} & \pi & 19.5 \end{bmatrix}$$

Size: 1×3

Introduction

- General From of a Matrix
  - Example Notations

$$\boldsymbol{A} = [a_{ij}] = \begin{bmatrix} 4\theta_1 & \theta_2 & -\frac{3}{L}d & \frac{wL^3}{24E_1} \\ \theta_1 & 4\theta_2 & -\frac{3}{L}d & \frac{-wL^3}{24E_1} \\ \theta_1 & \theta_2 & -\frac{4}{L}d & \frac{PL^2}{6EI} \end{bmatrix}$$

a_{22} a_{33} a_{14}

Formation of a Matrix

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- What Produces a Matrix?
  - Matrices are useful for representing sets of data.
  - The form of a matrix depends on the style or use.
  - The form may also be dictated by computational requirements.
  - The elements & variables can be properties that vary with time, space, or on the basis of any characteristic.

Formation of a Matrix

- What Produces a Matrix?
  - The following matrix is a data matrix of concentration of three water-quality indicators:

$$\begin{array}{c} \\ H \\ A \\ pH \end{array}\begin{array}{c} \begin{array}{ccc} \;\;H\;\; & \;\;A\;\; & \;\;pH\;\; \end{array} \\ \begin{bmatrix} 1.00 & -0.23 & -0.27 \\ -0.23 & 1.00 & 0.64 \\ -0.27 & 0.64 & 1.00 \end{bmatrix} \end{array}$$

H = hardness
A = alkalinity

Formation of a Matrix

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- Style and Use

5 × 3 matrix

$$\text{well}\;\begin{array}{c} \begin{array}{cccc} & H & A & pH \end{array} \\ \begin{bmatrix} 1 & 140 & 35 & 7.7 \\ 2 & 195 & 12 & 7.1 \\ 3 & 283 & 53 & 6.4 \\ 4 & 132 & 188 & 8.3 \\ 5 & 60 & 55 & 6.5 \end{bmatrix} \end{array}$$

3 × 5 matrix

$$\begin{array}{c} \\ \\ H \\ A \\ pH \end{array}\begin{array}{c} \text{well} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 140 & 195 & 283 & 132 & 60 \\ 35 & 12 & 53 & 188 & 55 \\ 7.7 & 7.1 & 6.4 & 8.3 & 6.5 \end{bmatrix} \end{array}$$

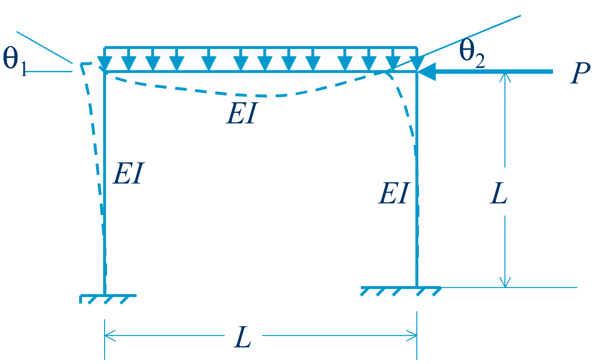
Formation of a Matrix

- What Produces a Matrix?
  - Results of matrix computations

$$\begin{bmatrix} 4 & 6 & 1 \\ 3 & 8 & 4 \end{bmatrix} \bullet \begin{bmatrix} 5 & 7 \\ 2 & 3 \\ 3 & 9 \end{bmatrix} = \begin{bmatrix} 35 & 55 \\ 43 & 81 \end{bmatrix}$$

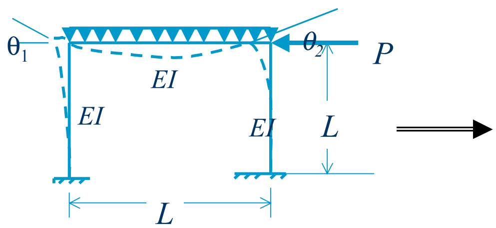
Formation of a Matrix

- What Produces a Matrix?
  - Result from physical structure of a problem



Formation of a Matrix

- What Produces a Matrix?
  - Result from physical structure of a problem



$$\begin{aligned} 4\theta_1 \quad \theta_2 \quad -\frac{3}{L}d &= \frac{wL^3}{24E} \\ \theta_1 \quad 4\theta_2 \quad -\frac{3}{L}d &= \frac{-wL^3}{24E} \\ \theta_1 \quad \theta_2 \quad -\frac{4}{L}d &= \frac{PL^2}{6EI} \end{aligned}$$

Formation of a Matrix

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- What Produces a Matrix?
  - Result from physical structure of a problem

$$\begin{aligned} 4\theta_1 \quad \theta_2 \quad -\frac{3}{L}d &= \frac{wL^3}{24E} \\ \theta_1 \quad 4\theta_2 \quad -\frac{3}{L}d &= \frac{-wL^3}{24E} \\ \theta_1 \quad \theta_2 \quad -\frac{4}{L}d &= \frac{PL^2}{6EI} \end{aligned} \quad \Longrightarrow \quad \begin{bmatrix} 4\theta_1 & \theta_2 & -\frac{3}{L}d & \frac{wL^3}{24E} \\ \theta_1 & 4\theta_2 & -\frac{3}{L}d & \frac{-wL^3}{24E} \\ \theta_1 & \theta_2 & -\frac{4}{L}d & \frac{PL^2}{6EI} \end{bmatrix}$$

Types of Matrices

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The following list provides different types of matrices:

1. Square Matrix
2. Upper Triangular Matrix
3. Strictly Upper Triangular Matrix
4. Lower Triangular Matrix
5. Strictly Lower Triangular Matrix
6. Diagonal Matrix

Types of Matrices

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7. Banded Matrix
8. Unit (identity)Matrix
9. Null Matrix
10. Symmetric Matrix
11. Skew-symmetric Matrix
12. Transposed Matrix

Types of Matrices

- Square Matrix
  - A *square matrix* is a special case in which the number of rows equals the number of columns
  - Examples

$$\begin{bmatrix} 4 & 1 & 1 \\ 3 & 2 & 1 \\ 0 & 4 & 2 \end{bmatrix} \qquad \begin{bmatrix} G & H \\ W & B \end{bmatrix} \qquad \begin{bmatrix} 4.76 \end{bmatrix}$$

Types of Matrices

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- Square Matrix
  - The set of elements a_{ij} for which $i = j$ is called the "**principal diagonal**"

$$\begin{bmatrix} 4 & 1 & 1 \\ 3 & 2 & 1 \\ 0 & 4 & 2 \end{bmatrix}$$

Types of Matrices

- Upper Triangular Matrix
  - An *upper triangular matrix* has values on the principal diagonal and above, but contains zero values for the matrix elements below the principal diagonal.
  - Examples:

$$\begin{bmatrix} 4 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \qquad \begin{bmatrix} -1 & 2 & 4 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Types of Matrices

- Strictly Upper Triangular Matrix
  - A *strictly upper triangular matrix* is an upper triangular matrix with the elements of the principal diagonal have zero values.
  - Examples:

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 2 & 4 & 1 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Types of Matrices

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- Lower Triangular Matrix
  - An *lower triangular matrix* has values on the principal diagonal and below, but contains zero values for the matrix elements above the principal diagonal.
  - Examples:

$$\begin{bmatrix} 4 & 0 & 0 \\ 3 & 2 & 0 \\ 7 & 4 & 2 \end{bmatrix} \qquad \begin{bmatrix} 5 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 2 & 1 & 4 & 1 \end{bmatrix}$$

Types of Matrices

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- Strictly Lower Triangular Matrix
  - A *strictly lower triangular matrix* is an lower triangular matrix with the elements of the principal diagonal have zero values.
  - Examples:

$$\begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 7 & 4 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & 4 & 0 \end{bmatrix}$$

Types of Matrices

- Diagonal Matrix
  - A *diagonal matrix* is a square matrix with all the elements equal to zero except for the elements on the principal diagonal.
  - Examples:

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1.2 & 0 \\ 0 & 0 & 7.1 \end{bmatrix} \qquad \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix}$$

Types of Matrices

- Banded Matrix
  - A *banded matrix* is a square matrix with all the elements equal to zero except for the principal diagonal and values in the positions adjacent to the principal diagonal.
    - A *tridiagonal matrix* is the especial case of a banded matrix that has zeros except in the principal diagonal and the two adjacent diagonals

Types of Matrices

– Examples: Banded Matrices

$$\begin{bmatrix} 1 & 3 & 0 & 0 & 0 \\ 2 & 7 & 4 & 0 & 0 \\ 0 & 3 & 8 & 3 & 0 \\ 0 & 0 & 1 & 10 & 5 \\ 0 & 0 & 0 & 6 & 15 \end{bmatrix}$$

A tridiagonal banded matrix

$$\begin{bmatrix} 0.5 & 2 & 7 & 0 & 0 & 0 \\ 2 & 1 & 3 & 2 & 0 & 0 \\ 7 & 6 & 2 & 5 & 1 & 0 \\ 0 & 1 & 2 & 4 & 9 & 3 \\ 0 & 0 & 4 & 2 & 5 & 2 \\ 0 & 0 & 0 & 6 & 9 & 7 \end{bmatrix}$$

A banded matrix