CHAPTER 1b: INTRODUCTION

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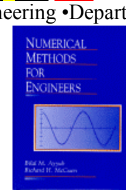

by

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ENCE 203 - Computation Methods in Civil Engineering II

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Numerical Analysis in Engineering

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- Course Objectives & Goals
  - Introduce numerical methods to engineering students and practicing engineers with the emphasis of their practical aspects in various engineering disciplines.

Numerical Analysis in Engineering

- Numerical Methods
  - The numerical methods deal with engineering problems that can be solved by hand or computers.
  - The numerical methods can be effectively demonstrated in cases dealing with complex problems for which analytical solutions cannot be obtained or hand calculations cannot be made.

Numerical Analysis in Engineering

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- Numerical Methods
  - In this course, common engineering problems are used to demonstrate the computational procedures.
  - The examples are intentionally selected with traceable solutions so that students can reproduce them.
  - Programming considerations for selected methods are provided.

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Numerical Analysis in Engineering

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- Numerical Methods
  – The use of any computational method, analytically or numerical, without the proper understanding of the limitations and shortcomings can have serious consequences.
  – When using numerical methods, the user should be aware of their:

Numerical Analysis in Engineering

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- Numerical Methods
  1. Computational details
  2. Their limitations
  3. Shortcomings, and
  4. Strength

  – Numerical methods should not be used as black boxes with input and output or as numerical recipes.

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Decision Making in Engineering

- Best Decisions
  - Full understanding of alternative solution procedures
    - Unbiased Solution
    - Highly precise
    - Cost effective
    - Have minimal environmental consequences

Decision Making in Engineering

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- Typical Approach to an Engineering Solution
  - Identify the problem
  - State the objectives & goals
  - Develop alternative solutions
  - Evaluate the alternatives, and
  - Use the best alternative

Decision Making in Engineering

- Numerical Methods, Probability, and Statistics
  - Knowledge of numerical methods probability, statistics, and reliability can help the engineer to ensure that each of the previously noted tasks are properly handled.

Engineering Design & Analysis

- Design of Engineering Systems
  - Design of engineering systems is usually a trade-off between maximizing safety and minimizing cost.
  - A design procedure that can accomplish both of these objective is highly desirable, but also difficult.

Expected Educational Outcomes

1. You should be able to numerically solve many types of problems such as:
   – Roots of equations
   – Systems of linear simultaneous equations
   – Interpolation of values of dependent variable given a set of discrete measurements
   – Approximating the differential or integral of unknown function given a set of discrete measurement from the function
   – Finding the solutions to differential equations

Expected Educational Outcomes

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2. The students should be able to select from alternative methods the one method that is most appropriate for a specific problem.
3. The students should be able to formulate algorithms to solve problems numerically.
4. They should understand the limitations of each numerical method, especially the conditions under which they fail to converge to a solution.

Analytical Versus Numerical Analysis

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- Analytical and Numerical approaches are distinguished on the basis of their algorithms
  - Analytical Calculus forms the basis fro analytical problem solving
  - Finite-difference arithmetic forms the basis of numerical methods

Analytical Versus Numerical Analysis

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- Analytical Methods
  - <u>Advantages and Disadvantages</u>
    - Analytical techniques provide direct solution and will result in an exact solution, if one exists.
    - Analytical methods usually require less time to find a solution.
    - Analytical solution procedure becomes considerably more complex when constraints are involved.

Analytical Versus Numerical Analysis

■ Numerical Methods

– Advantages and Disadvantages
  - Numerical techniques can be used for functions that have moderately complex structure.
  - It is easy to include constraints on the unknowns in the solution.
  - However, numerical methods require a considerable number of *iterations* in order to approach the true solution.
  - The solution usually is not exact, and therefore it is necessary to provide *initial* estimates of the unknowns

Analytical Versus Numerical Analysis

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■ Example: Analytical Vs. Numerical

Find the minimum of the following function both analytically and numerically:

$$y = x^2 - 3x + 2$$

Analytical Solution:

$$\frac{dy}{dx} = 2x - 3 = 0 \qquad \Rightarrow x = \frac{3}{2} = 1.5$$

Therefore, $y_{\min} = (1.5)^2 - 3(1.5) + 2 = -0.25$

Analytical Versus Numerical Analysis

- Example (cont'd): Analytical Vs. Numerical

  Numerical Solution:

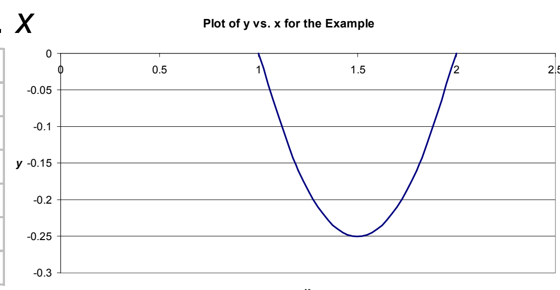
  One numerical solution is to iterate over a range of *x* values at a constant increment Δx and select the value of *x* for which *y* is smallest.

  For example, if the interval 1 to 2 is defined and an increment of 0.2 is used, the following Table and graph indicate that the minimum value of *y* falls in the interval $1.4 < x < 1.6$

Analytical Versus Numerical Analysis

- Table of *y* vs. *x*

x	*y*
1	0
1.2	-0.16
1.4	-0.24
1.6	-0.24
1.8	-0.16
2	0



$$y = x^2 - 3x + 2$$

Analytical Versus Numerical Analysis

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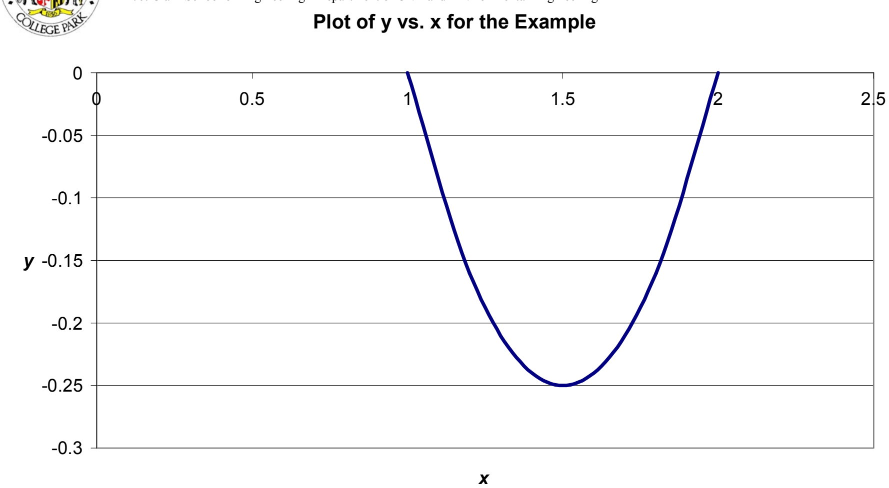

Analytical Versus Numerical Analysis

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- Example (cont'd): Analytical Vs. Numerical

  Numerical Solution:

  To improve the accuracy of the solution, the search interval of *x* could be established as $1.4 < x < 1.6$, the increment decreased to 0.02, and the search repeated as shown in the following viewgraph:

Analytical Versus Numerical Analysis

■ Example (cont'd): Analytical Vs. Numerical

$$y = x^2 - 3x + 2$$

x	*y*
1.4	-0.24
1.42	-0.2436
1.44	-0.2464
1.46	-0.2484
1.48	-0.2496
1.5	-0.25
1.52	-0.2496
1.54	-0.2484
1.56	-0.2464
1.58	-0.2436
1.6	-0.24

y_{min}

Taylor Series Expansion

■ Characteristics of Taylor Series

– The Taylor series is of great value in the study of numerical methods.

– In essence, the Taylor series provides a means to predict a function value at one point in terms of the function value and its derivatives at another point.

– A Taylor series is commonly used in engineering analysis to approximate functions that do not have closed form solution.

Taylor Series Expansion

- Characteristics of Taylor Series
  - A Taylor series is the sum of functions based on continually increasing derivatives.
  - For a function *f*(*x*) that depends on only one independent variable *x*, the value of the function at point $x_0 + h$ can be approximated by Taylor series.

Taylor Series Expansion

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- Taylor's Theorem
  - The Taylor series is given by

$$f(x_0+h)=f(x_0)+hf^{(1)}(x_0)+\frac{h^2}{2!}f^{(2)}(x_0)+\frac{h^3}{3!}f^{(3)}(x_0)+\ldots+\frac{h^n}{n!}f^{(n)}(x_0)+R_{n+1}$$

where

x_0 = base value or starting value

x = the point at which the value of the function is needed

$h = x - x_0$ = distance between x_0 and x (step size)

$n!$ = factorial of $n = n(n-1)(n-2)\ldots1$

$f^{(n)}$ = indicates the n^{th} derivative of the function $f(x)$

R_{n+1} = the remainder of Taylor series expansion

Taylor Series Expansion

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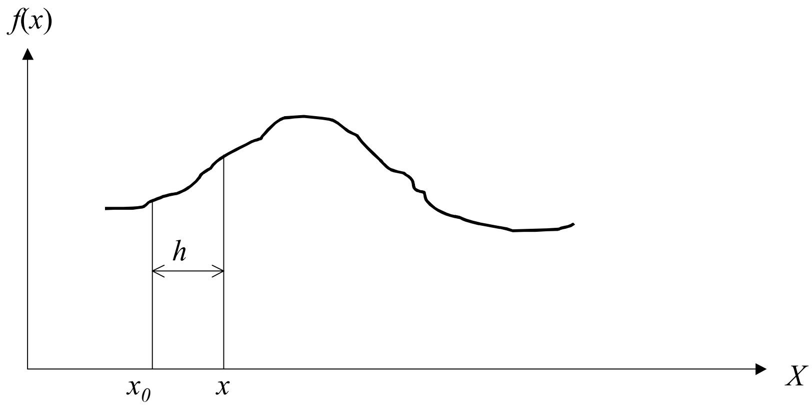

Taylor Series Expansion

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Taylor series expansion can also be given in a compact form as follows:

$$f(x_0 + h) = \sum_{k=0}^{\infty} \frac{h^k}{k!} f^{(k)}(x_0)$$

where

0! = 1 by convention

The above equation is based on the assumption that continuous derivative exist in an interval that include the points x_0 and x

Taylor Series Expansion

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- Order of Approximation
  – The order of the approximation is defined by the order of the highest derivative that is included in the approximation.
    • First-order approximation (two terms)

$$f(x_0+h)=f(x_0)+hf^{(1)}(x_0)$$

Taylor Series Expansion

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• Second-order approximation (three terms)

$$f(x_0+h)=f(x_0)+hf^{(1)}(x_0)+\frac{h^2}{2!}f^{(2)}(x_0)$$

• Third-order approximation (four terms)

$$f(x_0+h)=f(x_0)+hf^{(1)}(x_0)+\frac{h^2}{2!}f^{(2)}(x_0)+\frac{h^3}{3!}f^{(3)}(x_0)$$

Taylor Series Expansion

■ Example: Taylor Series

Develop a Taylor series expansion of the following function:

$$f(x) = x^5 - 3x^3 + 8$$

Use $x = x_0 = 3$ as the base (or starting) point and h as the increment. Evaluate the series for h = 0.2, 0.4, 0.6 to 4.0 and for 1,2,3,4,5 and 6 terms. Plot your results and compare them with the true value for each case.

Taylor Series Expansion

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■ Example: Taylor Series (cont'd)

$$f(x) = x^5 - 3x^3 + 8 \Rightarrow f(3) = 170$$

$x_0 = 3$

The function has the following derivatives:

$$f^{(1)}(x) = 5x^4 - 9x^2 \Rightarrow f^{(1)}(3) = 324$$
$$f^{(2)}(x) = 20x^3 - 18x \Rightarrow f^{(2)}(3) = 486$$
$$f^{(3)}(x) = 60x^2 - 18 \Rightarrow f^{(3)}(3) = 522$$
$$f^{(4)}(x) = 120x \Rightarrow f^{(4)}(3) = 360$$
$$f^{(5)}(x) = 120 \Rightarrow f^{(5)}(3) = 120$$
$$f^{(6)}(x) = 0$$

Taylor Series Expansion

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■ Example: Taylor Series (cont'd)

$$f(x_0+h)=f(x_0)+hf^{(1)}(x_0)+\frac{h^2}{2!}f^{(2)}(x_0)+\frac{h^3}{3!}f^{(3)}(x_0)+\ldots+\frac{h^n}{n!}f^{(n)}(x_0)+R_{n+1}$$

$$f(x)\approx f(x_0+h)=170+324h+486\frac{h^2}{2!}+522\frac{h^3}{3!}+360\frac{h^4}{4!}+120\frac{h^5}{5!}$$

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Taylor Series Expansion

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Example: Taylor Series

x	h	Constant $f(x_0+h)$ one term	Linear (1st order) $f(x_0+h)$ two terms	Quadratic (2nd order) $f(x_0+h)$ three terms	Cubic (3rd order) $f(x_0+h)$ four terms	(4th order) $f(x_0+h)$ five terms	Exact (5th order) $f(x_0+h)$ six terms	True Value
3.2	0.2	170	234.8	244.52	245.216	245.24	245.24032	245.24032
3.4	0.4	170	299.6	338.48	344.048	344.432	344.44224	344.44224
3.6	0.6	170	364.4	451.88	470.672	472.616	472.69376	472.69376
3.8	0.8	170	429.2	584.72	629.264	635.408	635.73568	635.73568
4	1	170	494	737	824	839	840.00000	840.00000
4.2	1.2	170	558.8	908.72	1059.056	1090.16	1092.64832	1092.64832
4.4	1.4	170	623.6	1099.88	1338.608	1396.232	1401.61024	1401.61024
4.6	1.6	170	688.4	1310.48	1666.832	1765.136	1775.62176	1775.62176
4.8	1.8	170	753.2	1540.52	2047.904	2205.368	2224.26368	2224.26368
5	2	170	818	1790	2486	2726	2758.00000	2758.00000
5.2	2.2	170	882.8	2058.92	2985.296	3336.68	3388.21632	3388.21632
5.4	2.4	170	947.6	2347.28	3549.968	4047.632	4127.25824	4127.25824
5.6	2.6	170	1012.4	2655.08	4184.192	4869.656	4988.46976	4988.46976
5.8	2.8	170	1077.2	2982.32	4892.144	5814.128	5986.23168	5986.23168
6	3	170	1142	3329	5678	6893	7136.00000	7136.00000
6.2	3.2	170	1206.8	3695.12	6545.936	8118.8	8454.34432	8454.34432
6.4	3.4	170	1271.6	4080.68	7500.128	9504.632	9958.98624	9958.98624
6.6	3.6	170	1336.4	4485.68	8544.752	11064.176	11668.8378	11668.8378
6.8	3.8	170	1401.2	4910.12	9683.984	12811.688	13604.0397	13604.0397
7	4	170	1466	5354	10922	14762	15786.0000	15786.0000

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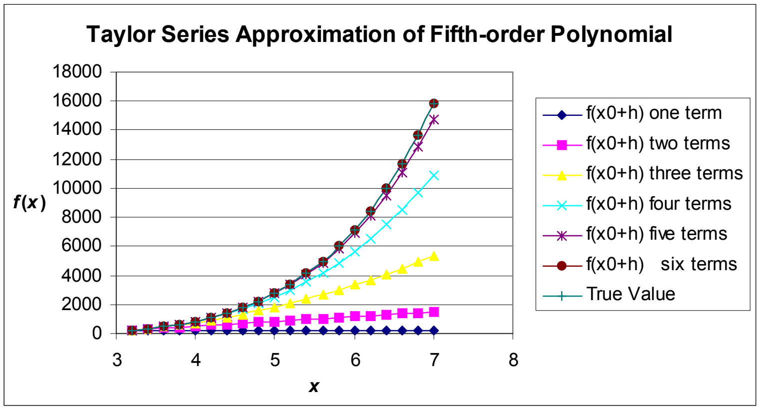
■ Example (cont'd): Taylor Series



Taylor Series Expansion

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■ Other Examples

These series can be used to evaluate their corresponding functions at any point x using a base value of $x_0 = 0$ and increment h.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + .. + \frac{x^k}{k!} = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ... = \sum_{k=0}^{\infty} x^k$$