CHAPTER 1a: INTRODUCTION

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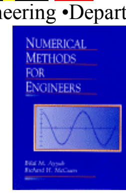

by

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ENCE 203 - Computation Methods in Civil Engineering II

Department of Civil and Environmental Engineering
University of Maryland, College Park

Background

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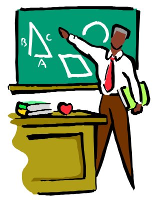
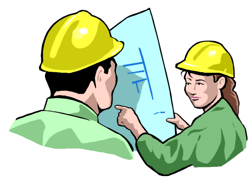
- The thinking of engineers toward mathematics has always been different from that of mathematicians.
  - Mathematician may be interested in finding out whether a solution to a differential equation exists.
  - An engineer simply assumes that the existence of a physical system is proof enough of the existence of a solution and focuses instead on finding it.

Background

- Attitude of a Mathematician and an Engineer toward Mathematics



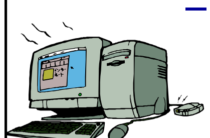
Mathematician



Engineers

Background

- Computers & Mathematical Tools

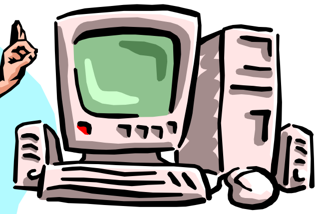


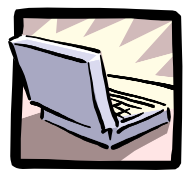
  - The availability of computers in all sizes at affordable costs has made a shift in the routine analysis and design methods used in engineering.

  - Analytical methods that were used in the past have been replaced in the analysis and design of actual systems by numerical techniques applied to more general models.

Background

- Computers & Mathematical Tools





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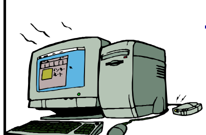

– These models are no longer restricted to those which have closed-form solution.
– These models that accurately represent realistically complex systems such as:
  - Nonlinear systems
  - Time-dependent state variable modeling techniques, and
  - Finite element analysis techniques
– The computer revolution has enabled engineers to successfully solve problems that were beyond their reach in the past.

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■ Mathematical Modeling and Engineering Problem Solving

- Knowledge and understanding are prerequisites for the effective implementation of any tool
- No matter how impressive your tool chest, you will be hard-pressed to repair a car if you do not understand how it works.

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- This specially true when using computers to solve engineering problems.
- Although they have powerful potential utility, computers are particularly useless without a fundamental understanding of how engineering systems work.

Background

■ The Engineering Problem Solving
- The understanding of engineering system is essentially gained by empirical means:
  - Observation
  - Experiment
- Over years of observation and experiment, engineers have noticed that certain aspects of their empirical studies occur repeatedly.

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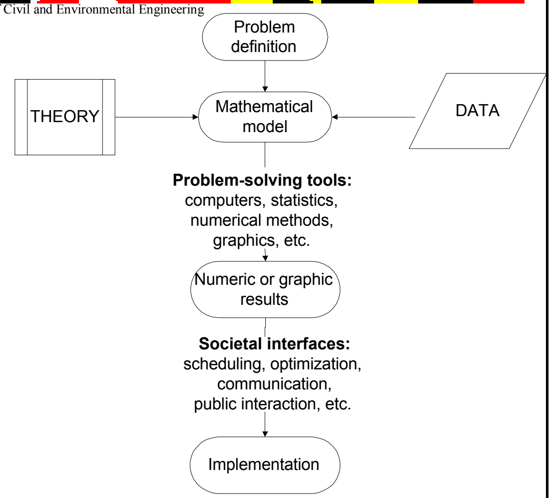
■ The Engineering Problem Solving
- Such general behavior can then be expressed as fundamental laws or models.
- These models essentially embody the cumulative wisdom of past experience.
- Most engineering problem solving uses the two-pronged approach of
  - empiricism
  - theory

Background

- The Engineering Problem Solving



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- The Engineering Problem Solving
  - Types of Mathematical Models
    - Simple
      - i.e., system of linear algebraic equations
    - Complex
      - i.e., partial differential equations in three spatial coordinates
  - In elementary calculus, you have learnt how to solve a variety of problems in a closed-form (so-called exact solutions).

Background

- The Engineering Problem Solving
  - Unfortunately, from a practical standpoint, the methods of calculus alone are not adequate to solve all the complex problems that an engineer might encounter.
  - In fact, you may have already faced problems in which an integral cannot be evaluated in a closed form such as

$$I = \int_a^b \left(e^{x^2} + \ln x\right) dx$$

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- The Engineering Problem Solving

$$I = \int_a^b f(x)dx = \text{area under the curve}$$

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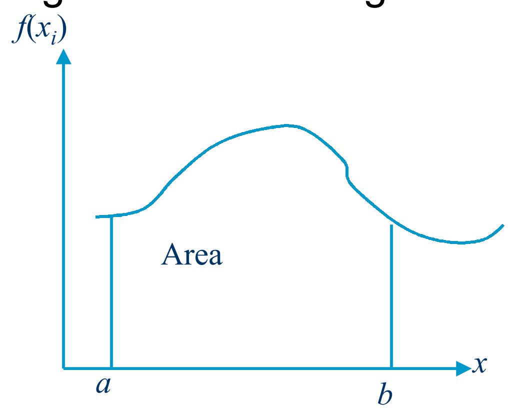

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■ The Engineering Problem Solving

$$I = \int_a^b f(x)dx = \text{area under the curve}$$

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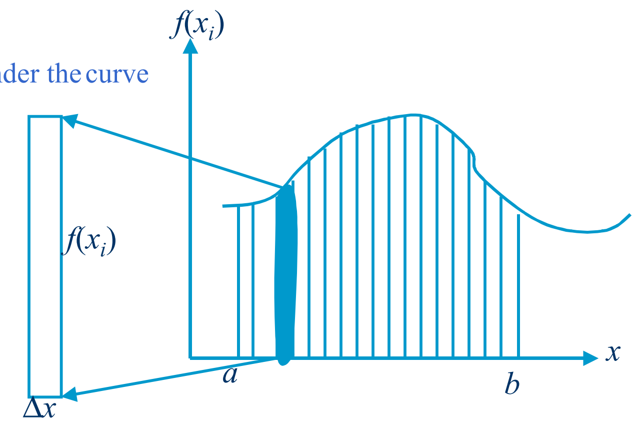
$$f(x) = e^{x^2} + \ln x$$



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■ The Engineering Problem Solving

$$I = \int_a^b f(x)dx = \text{area under the curve}$$

$$I = \int_a^b \left(e^{x^2} + \ln x\right)dx$$

$$f(x) = e^{x^2} + \ln x$$

Let $a = 1$ and $b = 10$, then

$$I = \int_0^{10} \left(e^{x^2} + \ln x\right)dx = \text{area} = \sum_{i=1}^{9} f(x_i)_i \Delta x_i$$

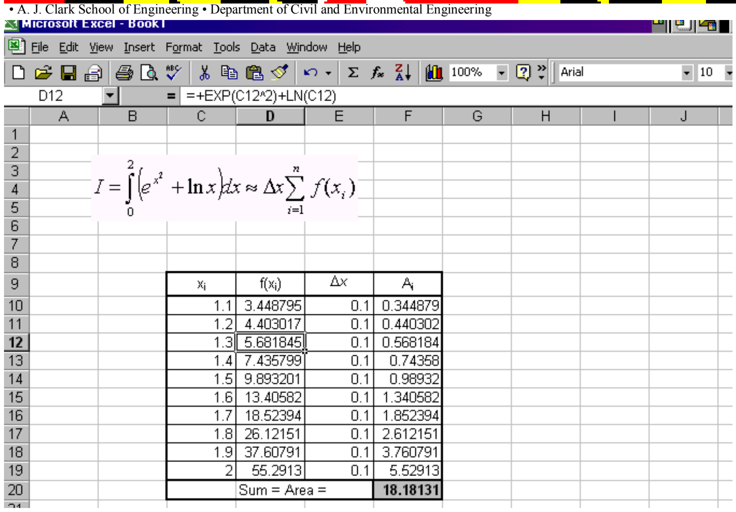
Background

■ The Engineering Problem Solving

$$I = \int_0^2 \left(e^{x^2} + \ln x\right) dx \approx \Delta x \sum_{i=1}^{n} f(x_i) = 18.18$$

x_i	$f(x_i)$	Δx	A_i
1.1	3.448795	0.1	0.344879
1.2	4.403017	0.1	0.440302
1.3	5.681845	0.1	0.568184
1.4	7.435799	0.1	0.74358
1.5	9.893201	0.1	0.98932
1.6	13.40582	0.1	1.340582
1.7	18.52394	0.1	1.852394
1.8	26.12151	0.1	2.612151
1.9	37.60791	0.1	3.760791
2	55.2913	0.1	5.52913
Sum = Area =			**18.18131**

Background



$$I = \int_0^2 \left(e^{x^2} + \ln x\right) dx \approx \Delta x \sum_{i=1}^{n} f(x_i)$$

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Background

■ A Simple Mathematical Model

“A *mathematical model* can be broadly defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms” (Chapra & Canale)

Background

■ Simple Mathematical Models

– Generally, a mathematical model can be represented as a functional relationship of the form

$$\begin{matrix}\text{Dependent} \\ \text{variable}\end{matrix} = f\left(\begin{matrix}\text{independent} \\ \text{variable}\end{matrix}, \text{parameters}, \begin{matrix}\text{forcing} \\ \text{functions}\end{matrix}\right)$$

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$$\begin{matrix}\text{Dependent}\\ \text{variable}\end{matrix} = f\begin{pmatrix}\text{independent} & & \text{forcing}\\ \text{variable} & \text{, parameters,} & \text{functions}\end{pmatrix}$$

Dependent variable =	A characteristic that usually reflects the behavior or state of the system
Independent variables =	Are usually dimensions, such as time and space
Parameters =	Are reflective of system's properties or compositions
Forcing functions =	Are external influences acting on the system

Background

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- Example: Newton's Second Law

$$F = ma \qquad \text{or} \qquad a = \frac{F}{m} \qquad (1)$$

F = force acting on the body (in newtons or kg-m per sec)
m = mass of the object (in kg)
a = acceleration of the object (meters per second squared)

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- Example: Newton's Second Law
  - Newton's second law equation has a number of characteristics that are typical of mathematical models of the physical world:
    1. It describes a natural process or system in mathematical terms
    2. It represents an idealization and simplification of reality. That is, the model ignores negligible details of the natural process and focuses on its essential part.

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- Example: Newton's Second Law

  Thus the second law does not include the effects of relativity that are of minimal importance when applied to objects and forces that interact on or about the earth's surface at velocities and on scales visible to humans.

  3. Finally, it yields reproducible results and , consequently, can be used for predictive purposes

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■ Example: Newton's Second Law

– Because of its simple algebraic form, the solution of Eq. 1 could be obtained easily.

– However, other mathematical models of physical phenomena may be much more complex, and either cannot be solved exactly or require more sophisticated mathematical techniques than simple algebra for their solutions

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■ More Complex Model

– Newton's Second Law

- To illustrate a more complex model, Newton's second law can be used to determine the terminal velocity of a free-falling body near the earth's surface.
- Let's assume that the falling body will be a parachutist.
- A model in this case can be derived as

$$m\frac{dv}{dt} = F$$

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- More Complex Model
  - Newton's Second Law

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- Newton's Second Law

$$m\frac{dv}{dt} = F$$

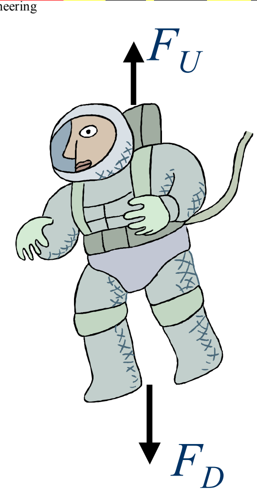
$F = F_D + F_U$

$F_D = mg$

$F_U = -cv$

F_D = downward force due to gravity

F_U = upward force due air resistance



Background

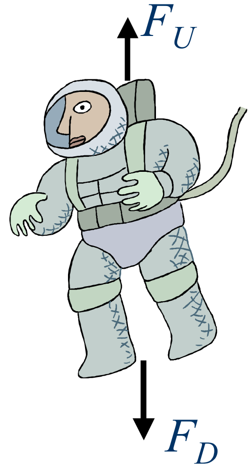
- Newton's Second Law

$$m\frac{dv}{dt} = F_D + F_U$$

$$= mg - cv$$

Or

$$\frac{dv}{dt} = g - \frac{c}{m}v$$



Background

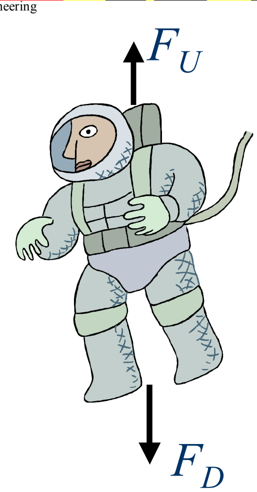
- Newton's Second Law

$$\boxed{\frac{dv}{dt} = g - \frac{c}{m}v} \qquad (2)$$

Calculus analytical solution
If $v = 0$ at $t = 0$

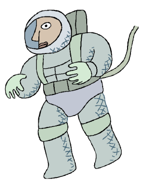
↓

$$\boxed{v(t) = \frac{gm}{c}\left[1 - e^{-(c/m)t}\right]} \qquad (3)$$



Background

- Example 1: Analytical Solution to the Falling Parachutist Problem



A parachutist with a mass of 68.1 kg jumps out of a stationary hot-air balloon. Compute the velocities *v* in an increment of 2 seconds prior to the opening of the chute. Use a drag coefficient value of 12.5 kg/s, and tabulate your values.

Background

- Example 1 (cont'd):

$$v = \frac{9.8(68.1)}{12.5}\left[1 - e^{-(12.5/68.1)t}\right]$$
$$= 53.3904\left(1 - e^{-0.18355t}\right)$$

t **(s)**	***v*** **(m/s)**
0	0
2	16.405
4	27.7693
6	35.6418
8	41.0953
10	44.8731
12	47.4902
∞	53.3904

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Microsoft Excel - Lecture 4 (2-5-01)

E13 = =+(9.8*68.1/12.5)*(1-EXP(-12.5*D13/68.1))

$$v(t) = \frac{gm}{c}\left[1 - e^{-(c/m)t}\right]$$

t (s)	*v* (m/s)
0	0
2	16.405
4	27.7693
6	35.6418
8	41.0953
10	44.8731
12	47.4902
∞	53.3904

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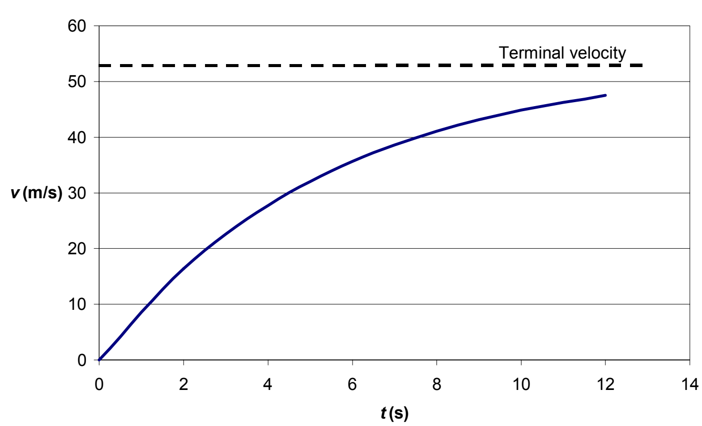
The analytical Solution to the falling parachutist problem



Background

- Numerical Solution to Newton's Second Law
  - As indicated earlier, numerical methods are those in which the mathematical problem is reformulated so it can be solved by arithmetic operations
  - This can be illustrated for Newton's second law by approximating the time rate of change of the velocity, that is

Background

- Numerical Solution to Newton's Second Law

$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

So, Eq. 2 becomes

$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c}{m} v$$

or

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c}{m} v(t_i) \right] (t_{i+1} - t_i) \qquad (4)$$

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- Numerical Solution to Newton's Second Law

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c}{m} v(t_i) \right] (t_{i+1} - t_i) \qquad (4)$$

New value = old value + slope X step size

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- Numerical Solution to Newton's Second Law
  - Thus the differential equation has been transformed into an equation that can be used to determine the velocity algebraically at t_{i+1} using the slope and previous values of *v* and *t.*
  - If you are given an initial value for the velocity at some time t_i, you can easily compute the velocity at a later time t_{i+1}.

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- Example 2: Numerical Solution to the Falling Parachutist Problem

  Perform the same computation as in Example 1 but use numerical methods (Eq. 4) to compute the velocities.

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c}{m} v(t_i) \right] (t_{i+1} - t_i)$$

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- Example 2 (cont'd):

  At t_i = 0, the velocity of the parachutist is zero. Using this information and the parameter values from Example 1, Eq. 4 can be used to compute the velocity at t_{i+1} = 2 seconds, that is

$$v = 0 + \left[9.8 - \frac{12.5}{68.1}(0) \right] (2 - 0) = 19.60 \text{ m/s}$$

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■ Example 2 (cont'd):

For the next interval (from t = 2 to 4 s), the computation is repeated, with the following result

$$v = 19.60 + \left[9.8 - \frac{12.5}{68.1}(19.60) \right](4-2) = 32.00 \text{ m/s}$$

The calculation is continued in a similar fashion to obtain additional values as shown the next viewgraph:

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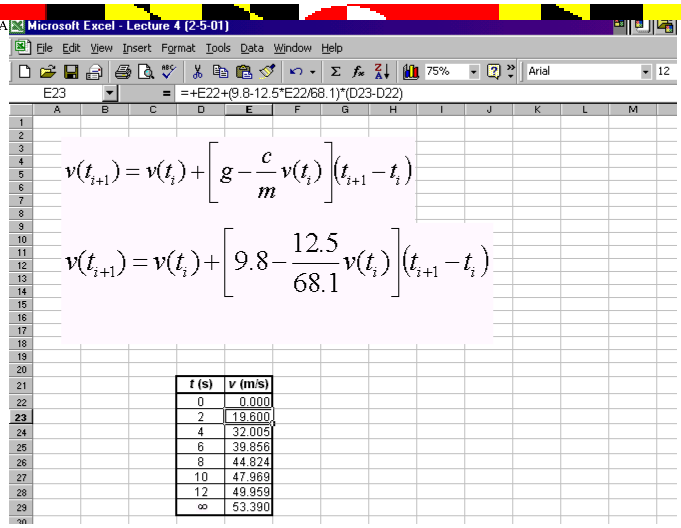
■ Example 2 (cont'd):

$$v(t_{i+1}) = v(t_i) + \left[9.8 - \frac{12.5}{68.1} v(t_i) \right](t_{i+1} - t_i)$$

t **(s)**	***v*** **(m/s)**
0	0.000
2	19.600
4	32.005
6	39.856
8	44.824
10	47.969
12	49.959
∞	53.390

Background

• A



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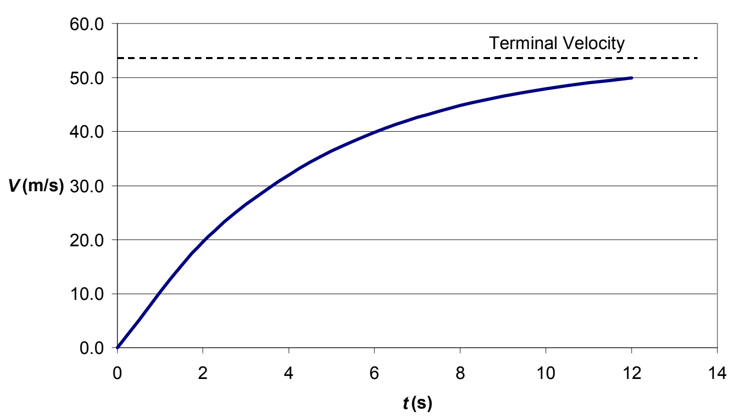
The Numerical Solution to the Falling Parachutist Problem as Computed in Example 2



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■ Comparison of Example 1 & 2

t (s)	Analytical *v* (m/s)	Numerical *v* (m/s)
0	0	0
2	16.4049808	19.6
4	27.76929146	32.00469897
6	35.64175156	39.8555437
8	41.09528323	44.82428683
10	44.87313757	47.96896861
12	47.49019095	49.95921508
∞	53.3904	53.3904

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Comparison of the numerical and analytical solutions for the falling parachutist problem