Homework #8 Solution
ENCE 203 - Spring 2001
Due M, 4/16

Problem1:

Textbook: 6-6

Let $F(x)$ be the function used to derive the data point

$$F(x) = a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots a_n x^n$$

Let $f(x)$ be the polynomial with the same order as the function $F(x)$

$$f(x) = b_0 + b_1 x + b_2 x^2 + \ldots\ldots\ldots b_n x^n$$

If the data set is given by the following table

x	c_1	c_2	………	c_n	c_{n+1}
$F(x)$	$f_1(x)$	$f_2(x)$	………	$f_n(x)$	$f_{n+1}(x)$

the method of undetermined coefficient will result in

$$\begin{aligned}
f_1(x) &= b_0 + b_1 c_1 + b_2 c_1^2 + \quad \ldots\ldots \quad + b_n c_1^n \\
f_2(x) &= b_0 + b_1 c_2 + b_2 c_2^2 + \quad \ldots\ldots \quad + b_n c_2^n \\
&\vdots \\
f_n(x) &= b_0 + b_1 c_n + b_2 c_n^2 + \quad \ldots\ldots \quad + b_n c_n^n \\
f_{n+1}(x) &= b_0 + b_1 c_{n+1} + b_2 c_{n+1}^2 + \quad \ldots\ldots \quad + b_n c_{n+1}^n
\end{aligned}$$

or, using a matrix format

$$\begin{bmatrix} 1 & c_1 & c_1^2 & \ldots\ldots & c_1^n \\ 1 & c_2 & c_2^2 & \ldots\ldots & c_2^n \\ . & . & . & . & . \\ . & . & . & . & . \\ 1 & c_n & c_n^2 & \ldots\ldots & c_n^n \\ 1 & c_{n+1} & c_{n+1}^2 & \ldots\ldots & c_{n+1}^n \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ . \\ . \\ b_2 \\ b_n \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ . \\ . \\ f_n(x) \\ f_{n+1}(x) \end{bmatrix} \qquad (1)$$

Since the data set is generated by the $F(x)$, the following matrix equation can be obtained

$$\begin{bmatrix} 1 & c_1 & c_1^2 & \ldots\ldots & c_1^n \\ 1 & c_2 & c_2^2 & \ldots\ldots & c_2^n \\ . & . & . & . & . \\ . & . & . & . & . \\ 1 & c_n & c_n^2 & \ldots\ldots & c_n^n \\ 1 & c_{n+1} & c_{n+1}^2 & \ldots\ldots & c_{n+1}^n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ . \\ . \\ a_2 \\ a_n \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ . \\ . \\ f_n(x) \\ f_{n+1}(x) \end{bmatrix} \qquad (2)$$

Comparing Eqs. (1) and (2) results in

$$\begin{bmatrix} a_0 \\ a_1 \\ . \\ . \\ a_2 \\ a_n \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ . \\ . \\ b_2 \\ b_n \end{bmatrix} \quad \Rightarrow \quad F(x) = f(x) \quad \Rightarrow$$ and the polynomial provide error free interpolated values

Problem 2:

Textbook: 6-18

(a) The following is the finite difference table:

Latitude(^{0}N)	R	ΔR	$\Delta^2 R$	$\Delta^3 R$	$\Delta^4 R$
0	891				
		-35			
20	856		-102		
		-137		14	
40	719		-88		24
		-225		38	
60	494		-50		
		-275			
80	219				

$$R(L) = 891 + n(-35) + \frac{n(n-1)}{2!}(-102) + \frac{n(n-1)(n-2)}{3!}(14) + \frac{n(n-1)(n-2)(n-3)}{4!}(24)$$

where $n = \dfrac{L - L_0}{\Delta L}$

(b) For $L = 35^0$, $L_0 = 0.0$, and $\Delta L = 20^0$

$$n = \frac{35 - 0.0}{20} = 1.75$$

$$R(35) = 891 + 1.75(-35) + \frac{1.75(1.75-1)}{2}(-102) + \frac{1.75(1.75-1)(1.75-2)}{6}(14)$$
$$+ \frac{1.75(1.75-1)(1.75-2)(1.75-3)}{24}(24)$$

$$= 762.5 \ \frac{\text{g.cal}}{\text{cm}^2} \ / \text{ day}$$

Problem 3:

Textbook: 6-25

$$L(D_0) = 100w_1 + 45w_2 + 39w_3 + 22w_4 + 5w_5 + 0.5w_6$$

where

$$w_1(D_0) = \frac{(D_0 - 1)(D_0 - 2)(D_0 - 10)(D_0 - 50)(D_0 - 100)}{(0-1)(0-2)(0-10)(0-50)(0-100)}$$

$$w_2(D_0) = \frac{(D_0 - 0)(D_0 - 2)(D_0 - 10)(D_0 - 50)(D_0 - 100)}{(1-0)(1-2)(1-10)(1-50)(1-100)}$$
$$w_3(D_0) = \frac{(D_0 - 0)(D_0 - 1)(D_0 - 10)(D_0 - 50)(D_0 - 100)}{(2-0)(2-1)(2-10)(2-50)(2-100)}$$
$$w_4(D_0) = \frac{(D_0 - 0)(D_0 - 1)(D_0 - 2)(D_0 - 50)(D_0 - 100)}{(10-0)(10-1)(10-2)(10-50)(10-100)}$$
$$w_5(D_0) = \frac{(D_0 - 0)(D_0 - 1)(D_0 - 2)(D_0 - 10)(D_0 - 100)}{(50-0)(50-1)(50-2)(50-10)(50-100)}$$
$$w_6(D_0) = \frac{(D_0 - 0)(D_0 - 1)(D_0 - 2)(D_0 - 10)(D_0 - 50)}{(100-0)(100-1)(100-2)(100-10)(100-50)}$$

For D_0 = 1.6 m

$$\begin{aligned} L(1.6) &= 100(-0.0960) + 45(0.5864) + 39(0.5103) + 22(-0.00071) + 5(1.3495*10^{-6}) \\ &\quad + 0.5(-3.5759*10^{-8}) \\ &= 36.674\ \% \end{aligned}$$

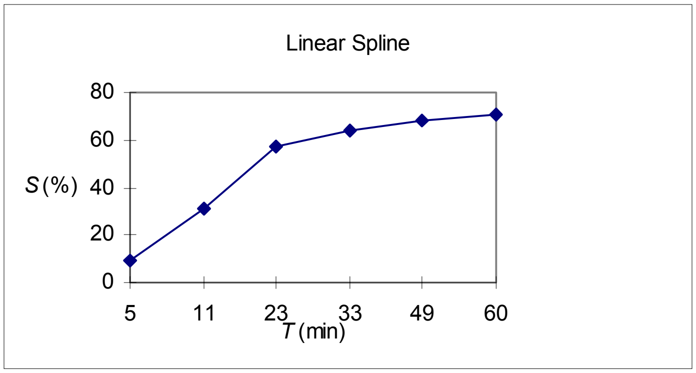
Problem 4:

Textbook: 6-30

T (min)	5	11	23	33	49	60
S (%)	9	31	57	64	68.00	71

Linear Spline:

T_i (min)	$S(T_i)$ (%)	Linear Spline $S_i(T)$
5	9	
		$9 - \frac{31-9}{11-5}(T-5) = 9 + 3.67(T-5)$
11	31	
		$31 - \frac{57-31}{23-11}(T-11) = 31 + 2.17(T-11)$
23	57	
		$57 - \frac{64-57}{33-23}(T-23) = 57 + 0.70(T-23)$
33	64	
		$64 - \frac{68-64}{49-33}(T-33) = 64 + 0.25(T-33)$
49	68	
		$68 - \frac{71-68}{60-49}(T-49) = 68 + 0.273(T-49)$
60	71	



Quadratic Spline:

Using Eq. 6-71a of the textbook, the following equations can be developed:

$$\begin{aligned}
25\,a_1 + 5\,b_1 + c_1 &= 9 \\
121\,a_2 + 11\,b_2 + c_2 &= 31 \\
529\,a_3 + 23\,b_3 + c_3 &= 57 \\
1089\,a_4 + 33\,b_4 + c_4 &= 64 \\
2401\,a_5 + 49\,b_5 + c_5 &= 68
\end{aligned}$$

Equation 6-71b of the textbook results in the following conditions:

$$\begin{aligned}
121\,a_1 + 11\,b_1 + c_1 &= 31 \\
529\,a_2 + 23\,b_2 + c_2 &= 57 \\
1089\,a_3 + 33\,b_3 + c_3 &= 64 \\
2401\,a_4 + 49\,b_4 + c_4 &= 68 \\
3600\,a_5 + 60\,b_5 + c_5 &= 71
\end{aligned}$$

Equation 6-72 of the textbook results in the following conditions:

$$\begin{aligned}
2a_1(11) + b_1 &= 2a_2(11) + b_2 \\
2a_2(23) + b_2 &= 2a_3(23) + b_3 \\
2a_3(33) + b_3 &= 2a_4(33) + b_4 \\
2a_4(49) + b_4 &= 2a_5(49) + b_5
\end{aligned}$$

or

$$\begin{aligned}
22\,a_1 + b_1 &= 22\,a_2 + b_2 \\
46\,a_2 + b_2 &= 46\,a_3 + b_3 \\
66\,a_3 + b_3 &= 66\,a_4 + b_4 \\
98\,a_4 + b_4 &= 98\,a_5 + b_5
\end{aligned}$$

Since $a_1 = 0$, the resulting system of 14 equations can be summarized in a matrix format as

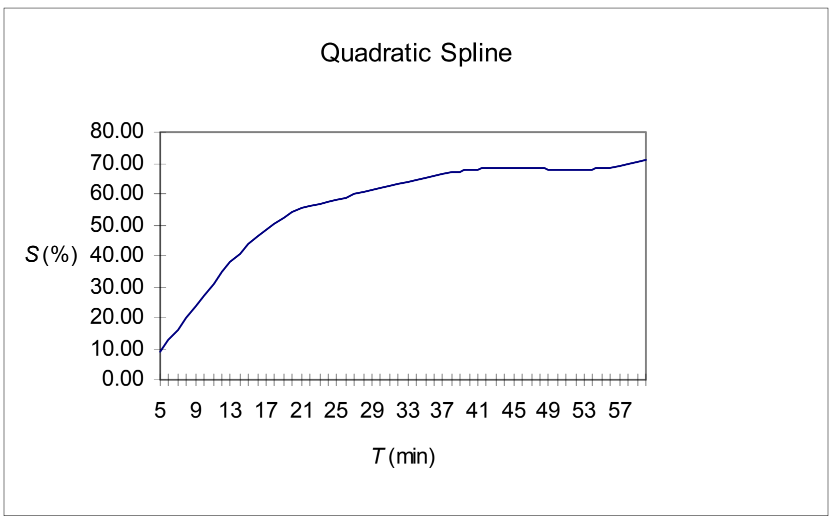
$$\begin{bmatrix} 5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 121 & 11 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 529 & 23 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1089 & 33 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2401 & 49 & 1 \\ 11 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 529 & 23 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1089 & 33 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2401 & 49 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3600 & 60 & 1 \\ 1 & 0 & -22 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 46 & 1 & 0 & -46 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 66 & 1 & 0 & -66 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 98 & 1 & 0 & -98 & -1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \\ a_4 \\ b_4 \\ c_4 \\ a_5 \\ b_5 \\ c_5 \end{bmatrix} = \begin{bmatrix} 9 \\ 31 \\ 57 \\ 64 \\ 68 \\ 31 \\ 57 \\ 64 \\ 68 \\ 71 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The solution of this system of equations is

$$\begin{bmatrix} b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \\ a_4 \\ b_4 \\ c_4 \\ a_5 \\ b_5 \\ c_5 \end{bmatrix} = \begin{bmatrix} 3.667 \\ -9.333 \\ -0.125 \\ 6.417 \\ -24.458 \\ 0.0033 \\ 0.5133 \\ 43.430 \\ -0.0302 \\ 2.727 \\ 6.903 \\ 0.0460 \\ -4.742 \\ 189.893 \end{bmatrix}$$

The following table summarizes the Spline:

T_i (min)	$S(T_i)$ (%)	Quadratic Spline $S_i(T)$
5	9	
		$3.667T - 9.333$
11	31	
		$-0.125T^2 + 6.417T - 24.458$
23	57	
		$0.00333T^2 + 0.5133T + 43.43$
33	64	
		$-0.0302T^2 + 2.727T + 6.903$
49	68	
		$-0.046T^2 - 4.742T + 189.893$
60	71	



Problem 5:

Textbook: 6-31

(a). $$f(2,22.3) = 0.8123 + \frac{0.9610 - 0.8123}{30 - 20}(22.3 - 20) = 0.846501$$

$$f(3,22.3) = 2.1670 + \frac{2.8901 - 2.1670}{30 - 20}(22.3 - 20) = 2.333313$$

$$f(2.3,22.3) = 0.846501 + \frac{2.333313 - 0.846501}{3 - 2}(2.3 - 2) = 1.2925446$$

(b). $$f(3,10.6) = 0.8190 + \frac{2.1670 - 0.8190}{20 - 10}(10.6 - 10) = 0.89988$$

$$f(4,10.6) = 1.8231 + \frac{3.1621 - 1.8231}{20 - 10}(10.6 - 10) = 1.90344$$

$$f(3.4,10.6) = 0.89988 + \frac{1.90344 - 0.89988}{4 - 3}(3.4 - 3) = 1.301304$$