Homework #5 Solution
ENCE 203 - Spring 2001
Due F, 3/9

Problem1:

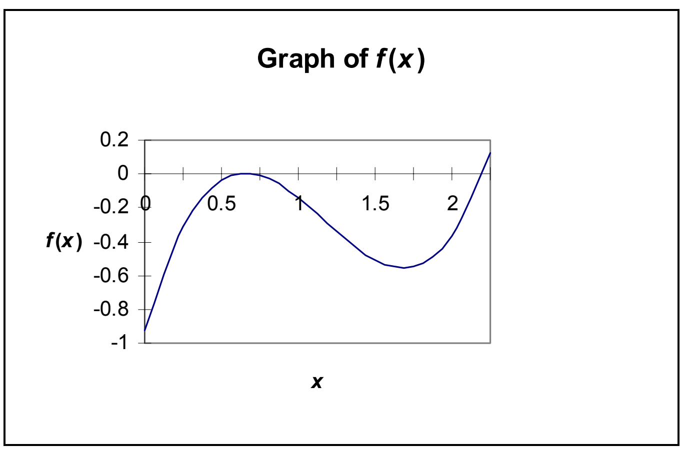
Textbook: 4-5

(a)

x	$f(x)$
0	-0.924
0.25	-0.30713
0.5	-0.034
0.75	-0.01088
1	-0.144
1.25	-0.33963
1.5	-0.504
1.75	-0.54338
2	-0.364
2.25	0.12788

The above table gives the value of x and $f(x)$ using the direct-search method with an increment of 0.25. The table shows that there exists one root in the interval (2, 2.25). The function changes sign in this interval. In fact, the function has three real roots. These roots are 2.2, 0.6, and 0.7. The direct-search method only identified one root and already missed two, and probably it will not find them unless the increment is reduced.

(b)



The most feasible intervals to search for roots are (0.50, 0.75) and (2.0, 2.25).

Problem 2:

Textbook: 4-8

(a). $$f(x) = x^4 - 1.74x^3 + x^2 - 0.4252x + 0.0027 = 0$$

From the figure of Problem 4-6, we know that there is a root within the interval (0, 0.02), hence for i = 0, $x_s = 0$, and $x_e = 0.02$. Therefore,

$$x_m = \frac{x_s + x_e}{2} = \frac{0 + 0.02}{2} = 0.01$$

$$f(x_s) = 0.0027$$

$$f(x_e) = -0.005441$$

$$f(x_m) = -0.0014595$$

$$f(x_s)\,.\,f(x_m) = \text{negative}$$

$$f(x_m)\,.\,f(x_e) = \text{positive}$$

The root lies between 0 and 0.01

Estimate of the root = $x_m = 0.01$

For i = 1, $x_s = 0$, and $x_e = 0.01$,

$$x_m = \frac{x_s + x_e}{2} = \frac{0 + 0.01}{2} = 0.005$$

$$f(x_s) = 0.0027$$

$$f(x_e) = -0.0014595$$

$$f(x_m) = 0.00059733$$

$$f(x_s)\,.\,f(x_m) = \text{positive}$$

$$f(x_m)\,.\,f(x_e) = \text{negative}$$

The root lies between 0.005 and 0.01

Estimate of the root = 0.005

$$\varepsilon_d = |0.005 - 0.01| = 0.005 > 0.002$$

Since the criterion for the tolerance of 0.002 is not met, we continue the iterative procedure until the required tolerance is satisfied. The following table summarizes the results. The first root was found to be 0.0063 (to 4 significant figures).

i	x_s	x_m	x_e	$f(x_s)$	$f(x_m)$	$f(x_e)$	$f(x_s).f(x_m)$	$f(x_m).f(x_e)$	ε_d
0	0.0000	0.0100	0.0200	0.0027	-0.001	-0.005	-	+	----
1	0.0000	0.0050	0.0100	0.003	0.001	-0.001	+	-	0.005
2	0.0050	0.0075	0.0100	0.001	-4E-04	-0.001	-	+	0.003
3	0.0050	0.0063	0.0075	0.001	8E-05	-4E-04	+	-	0.001

(b) Same as part (a) except that the error was computed as relative percent error as follows:

$$\varepsilon_r = \left|\frac{0.005 - 0.01}{0.005}\right| \text{ x } 100 = 100\ \% > 0.1\ \%$$

Since the criterion for the tolerance of 0.1% is not met, we continue the iterative procedure until the required tolerance is satisfied. The following table summarizes the results. The first root was found to be 0.0064 (to 4 significant figures).

i	x_s	x_m	x_e	$f(x_s)$	$f(x_m)$	$f(x_e)$	$f(x_s).f(x_m)$	$f(x_m).f(x_e)$	$\varepsilon_{r(\%)}$
0	0.0000	0.0100	0.0200	0.0027	-0.0015	-0.0054	-	+	----
1	0.0000	0.0050	0.0100	0.0027	0.0006	-0.0015	+	-	100.0
2	0.0050	0.0075	0.0100	0.0006	-0.0004	-0.0015	-	+	33.3
3	0.0050	0.0063	0.0075	0.0006	0.0001	-0.0004	+	-	20.0
4	0.0063	0.0069	0.0075	0.0001	-0.0002	-0.0004	-	+	9.1
5	0.0063	0.0066	0.0069	0.0001	-0.0001	-0.0002	-	+	4.8
6	0.0063	0.0064	0.0066	8E-05	1E-05	-5E-05	+	-	2.4
7	0.0064	0.0065	0.0066	1E-05	-2E-05	-5E-05	-	+	1.2
8	0.0064	0.0064	0.0065	1E-05	-2E-06	-2E-05	-	+	0.6
9	0.0064	0.0064	0.0064	1E-05	6E-06	-2E-06	+	-	0.3
10	0.0064	0.0064	0.0064	6E-06	2E-06	-2E-06	+	-	0.2
11	0.0064	0.0064	0.0064	2E-06	1E-07	-2E-06	+	-	0.1

Problem 3:

Textbook: 4-13

$$f(x) = x^3 - 6.1\,x^2 + 11.26x - 6.336$$

$$f'(x) = 3x^2 - 12.2x + 11.26$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	ε_r (%)
0	1	-0.176	2.06	1.08544	-----
1	1.0854	-0.022	1.55219	1.0996	7.9
2	1.0996	-0.00057	1.47217	1.1000	1.3

From the above table, the first root is 1.1. Therefore,

$$x^2 - 5x + 5.76$$
$$x - 1.1 \mid x^3 - 6.1x^2 + 11.26x - 6.336$$
$$x^3 - 1.1x^2$$
$$- 5.0x^2 + 11.26x - 6.336$$
$$- 5.0x^2 + 5.50x$$
$$5.76x - 6.336$$
$$5.76 - 6.336$$
$$0 = \text{error}$$

The Newton-Raphson method can be used now to find a root for the reduced polynomial

$$x^2 - 5x + 5.76$$

The following table summarizes the results:

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	ε_r (%)
0	1	1.76	-3	1.58667	-----
1	1.5867	0.34418	-1.82667	1.7751	37.0
2	1.7751	0.0355	-1.44983	1.7996	10.6
3	1.7996	0.0006	-1.40086	1.8000	1.4

From the above table, a second root of 1.8 is found. Using the polynomial reduction technique once more, the third root can be found as follows:

$$
\begin{array}{r|l}
 & x - 3.2 \\ \hline
x - 1.8 & x^2 - 5x + 5.76 \\
 & \underline{x^2 - 1.8x} \\
 & \quad -3.2x + 5.76 \\
 & \quad \underline{-3.2x + 5.76} \\
 & \qquad 0 = \text{error}
\end{array}
$$

Summary of roots:

$x_1 =$ 1.1
$x_2 =$ 1.8
$x_3 =$ 3.2

Problem 4:

Use the secant method to estimate the root of $f(x) = e^{-x} - x$. Start with initial estimates of $x_0 = 0$ and $x_1 = 1.0$, and a precision (tolerance) of 5%.

***** SOLUTION *****

First Iteration: $i = 1$:

$$x_0 = 0 \Rightarrow f(0) = e^{-(0)} - (0) = 1$$

$$x_1 = 1 \Rightarrow f(1) = e^{-(1)} - 1 = -0.63212$$

$$x_2 = x_1 - \frac{f(x_1)[x_0 - x_1]}{f(x_0) - f(x_1)} = 1 - \frac{-0.63212[0-1]}{1-(-0.63212)} = 0.61270$$

First Iteration: $i = 2$:

$$x_1 = 1, \Rightarrow f(x_1) = -0.63212$$

$$x_2 = 0.61270, \Rightarrow f(0.61270) = -0.07081$$

$$x_3 = x_2 - \frac{f(x_2)[x_1 - x_2]}{f(x_1) - f(x_2)} = 0.61270 - \frac{-0.07081[1 - 0.61270]}{-0.63212 - (-0.07081)} = 0.56384$$

First Iteration: $i = 3$:

$$x_2 = 0.61270, \Rightarrow f(x_1) = -0.07081$$

$$x_3 = 0.56384, \Rightarrow f(0.56384) = 0.00518$$

$$x_4 = x_3 - \frac{f(x_3)[x_2 - x_3]}{f(x_2) - f(x_3)} = 0.56384 - \frac{0.00518[0.61270 - 0.56384]}{-0.07081 - 0.00518} = 0.56717$$

$$f(0.56717) = -0.00004$$

Hence, the root is 0.56717 to 4 significant digits

Problem 5:

Textbook: 4-28

$$f(x) = x^3 + 0.9x^2 - 10.56x - 16.94$$

$$x_{i+1} = x_i - \frac{R_0(x_i)}{R_1(x_i)}$$

i	0	1	2	3	4	5	6	7
x_i	1.00	-3.444	-2.873	-2.554	-2.382	-2.292	-2.247	-2.223
ε_r(%)	---	129.0	19.9	12.5	7.2	3.9	2.0	
R_0	-25.60	-10.75	-2.89	-0.757	-0.194	-0.049	-0.012	
R_1	-5.76	18.833	9.037	4.406	2.1718	1.0776	0.5367	
b_3	1.00	1	1	1	1	1	1	
b_2	0.90	0.9	0.9	0.9	0.9	0.9	0.9	
b_1	-10.56	-10.56	-10.56	-10.56	-10.56	-10.56	-10.56	
b_0	-16.94	-16.94	-16.94	-16.94	-16.94	-16.94	-16.94	
c_3	1.00	1	1	1	1	1	1	
c_2	1.90	-2.544	-1.973	-1.654	-1.482	-1.392	-1.347	
c_1	-8.66	-1.796	-4.89	-6.337	-7.031	-7.368	-7.535	
d_3	1.00	1	1	1	1	1	1	
d_2	2.90	-5.989	-4.847	-4.207	-3.864	-3.685	-3.593	
d_1	-5.76	18.833	9.037	4.406	2.1718	1.0776	0.5367	

From the above table, the first root is -2.223. Using synthetic division for the second order polynomial x^2 -1.347x -7.535, the following table summarizes the results:

i	0	1	2	3	4	5	6	7
x_i	1	13.061	7.1894	4.5443	3.6406	3.503	3.4996	3.4996
ε_r(%)	---	92.3	81.7	58.2	24.8	3.9	0.1	
R_0	-7.882	145.46	34.472	6.9967	0.8167	0.0189	1E-05	
R_1	0.6535	24.775	13.032	7.7421	5.9347	5.6594	5.6527	
b_2	1	1	1	1	1	1	1	
b_1	-1.347	-1.347	-1.347	-1.347	-1.347	-1.347	-1.347	
b_0	-7.535	-7.535	-7.535	-7.535	-7.535	-7.535	-7.535	
c_2	1	1	1	1	1	1	1	
c_1	-0.347	11.714	5.8429	3.1978	2.2941	2.1565	2.1531	
d_2	1	1	1	1	1	1	1	
d_1	0.6535	24.775	13.032	7.7421	5.9347	5.6594	5.6527	

From the above table, a second root of 3.4996 = 3.5 is found. The values c_2 = 1, and c_1 = 2.1531 $\cong$ 2.2 give the polynomial

$$x + 2.2 = 0$$

Thus, the third root is -2.2.

<u>Summary of roots:</u>

$x_1 =$ -2.2
$x_2 =$ 3.5
$x_3 =$ -2.2

<u>Problem 6:</u>

Textbook: 4-38

$$\begin{vmatrix} 2.0-\lambda & 1.2 & 0.8 \\ 0.6 & 3.0-\lambda & 1.5 \\ 1.1 & 1.4 & 1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^3 - 6\lambda^2 + 7.3\lambda - 1.092 = 0 \Rightarrow \lambda_1 = 4.396,\ \lambda_2 = 0.174,\ \lambda_3 = 1.430$$