Homework #4 Solution
ENCE 203 - Spring 2001
Due F, 3/2

Problem1:

Textbook: 2-25

The normal vector condition results in

$a_{12}^2 + a_{32}^2 = 1-(0.6)^2 = 0.64 \quad (1)$

The vector orthogonality condition results in

$0.5a_{12} - 0.843a_{32} = 0.12 \quad (2)$

From (1) and (2),

$a_{12} = 0.742428, -0.617512$

and

$a_{32} = 0.298000, -0.508607$

Hence, there are two pairs of values for a_{12} and a_{32} that are necessary for matrix ***A*** to be orthonormal in the columns. These pairs are:

$$\begin{bmatrix} a_{12} \\ a_{32} \end{bmatrix} = \begin{bmatrix} 0.742428 \\ 0.298000 \end{bmatrix}$$

and

$$\begin{bmatrix} a_{12} \\ a_{32} \end{bmatrix} = \begin{bmatrix} -0.617512 \\ -0.508607 \end{bmatrix}$$

Problem 2:

Textbook: 2-28

$$\begin{vmatrix} -2 & 6 & 2 \\ 1 & -3 & 2 \\ 2 & -6 & -2 \end{vmatrix} = -2\,[\,(-3 \text{ x } -2) - (2 \text{ x } -6)\,] - 6\,[\,(1 \text{ x } -2) - (2 \text{ x } 2)\,] + 2\,[\,(1 \text{ x } -6) - (-3 \text{ x } 2)\,] = 0.0$$

Problem 3:

Textbook: 2-30

a. $r(\begin{bmatrix} 1 & 2 & 4 \\ 1.5 & 4 & 6 \end{bmatrix}) = 2$, because $\begin{vmatrix} 2 & 4 \\ 4 & 6 \end{vmatrix} = -4 \neq 0$

b. $r(\begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix}) = 3$, because $\begin{vmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \end{vmatrix} = 18 \neq 0$

c. $r(\begin{bmatrix} 2.0 & 8.0 & 8.0 \\ 1.5 & 4.5 & 6.0 \\ 3.5 & 24.5 & 14.0 \\ 1.5 & 4.5 & 6.0 \end{bmatrix}) = 2$ because of the following

All combination of 3 by 3 matrices have a determinant equal to zero. The second combination of matrices that need to be tested is the 2 by 2 . At least one of the 2 by 2 matrices produces a determinant of -3, namely

$$\begin{bmatrix} 2.0 & 8.0 \\ 1.5 & 4.5 \end{bmatrix}$$

Hence, the rank = 2.

Problem 4:

Using the definition of the inverse as given by $A^{-1} = \dfrac{A^a}{|A|}$ in your notes, find the inverse of the following matrix ***A***:

$$\boldsymbol{A} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 0 & 1 \\ 3 & 9 & 2 \end{bmatrix}$$

*** SOLUTION ***

$$\det(A) = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 0 & 1 \\ 3 & 9 & 2 \end{vmatrix} = -2[(2)(4) - (1)(3)] - 9[(3)(1) - (1)(4)] = -2(5) - 9(-1) = -1$$

$$A^c = \begin{bmatrix} \begin{vmatrix} 0 & 1 \\ 9 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 4 & 0 \\ 3 & 9 \end{vmatrix} \\ -\begin{vmatrix} 2 & 1 \\ 9 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 3 & 9 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 4 & 0 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -9 & -5 & 36 \\ 5 & 3 & -21 \\ 2 & 1 & -8 \end{bmatrix}$$

$$A^a = \left(A^c\right)^T = \begin{bmatrix} -9 & 5 & 2 \\ -5 & 3 & 1 \\ 36 & -21 & -8 \end{bmatrix}$$

Therefore,

$$A^{-1} = \frac{A^a}{|A|} = \frac{\begin{bmatrix} -9 & 5 & 2 \\ -5 & 3 & 1 \\ 36 & -21 & -8 \end{bmatrix}}{-1} = \begin{bmatrix} 9 & -5 & -2 \\ 5 & -3 & -1 \\ -36 & 21 & 8 \end{bmatrix}$$

Problem 5:

Textbook: 3-1

$x_0 + \Delta x = \sqrt[3]{x}$, or $(x_0 + \Delta x)^3 = x$; neglecting the term that involve $(\Delta x)^2$ or higher, we get:

$x_0^3 + 3x^2\,\Delta x = x,$ or $\Delta x = \dfrac{x - x_0^3}{3x_0^2}$, In general :

$$\Delta x = \frac{x - x_i^3}{3x_i^2}$$

If $x = 31$, and $x_0 = 3$, then $\Delta x = \dfrac{31 - 3^3}{3(3)^2} = 0.148148$

with $\Delta x = 3 + 0.148148 = 3.148148$, $\Delta x = \dfrac{31 - (3.148148)^3}{3(3.148148)^2} = -0.006753$

A third iteration will give $\Delta x = -0.000015$ and $x_3 = 3.141381$, which is within the desired accuracy of 0.00005.

Problem 6:

Textbook: 3-7

$V = Lwh = (3.21\underline{7})(0.792\underline{4})(1.30\underline{2}) = 3.31\underline{9}$ (4 significant figure)