

Homework #3 Solution  
 ENCE 203 - Spring 2001  
 Due F, 2/23

**Problem 1:**

Textbook: 2-5

$$\text{a) } \mathbf{A}^T = \begin{bmatrix} 2 & -3 \\ 6 & 4 \end{bmatrix}, \quad \text{b) } \mathbf{B}^T = \begin{bmatrix} -1 & 3 \\ -9 & 0 \\ 6 & -7 \end{bmatrix}, \quad \text{c) } \mathbf{C}^T = \begin{bmatrix} 0.2 & -0.8 & -0.3 \\ -0.5 & 0.1 & 0.7 \end{bmatrix}$$

**Problem 2:**

Textbook: 2-6

Addition:

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 3-2 & -2+3 \\ 0+1 & 4+2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 6 \end{bmatrix}, \quad \mathbf{B} + \mathbf{A} = \begin{bmatrix} -2+3 & 3+-2 \\ 1+0 & 2+4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 6 \end{bmatrix}$$

Hence,  $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \implies$  and matrix addition is not directional

Subtraction:

$$\mathbf{A} - \mathbf{B} = \begin{bmatrix} 3-(-2) & -2-3 \\ 0-1 & 4-2 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ -1 & 2 \end{bmatrix}, \quad \mathbf{B} - \mathbf{A} = \begin{bmatrix} -2-3 & 3-(-2) \\ 1-0 & 2-4 \end{bmatrix} = \begin{bmatrix} -5 & 5 \\ 1 & -2 \end{bmatrix}$$

Hence,  $\mathbf{A} - \mathbf{B} \neq \mathbf{B} - \mathbf{A} \implies$  and matrix subtraction is directional

**Problem 3:**

Textbook: 2-9

$$\mathbf{F} \cdot \mathbf{G} = \begin{bmatrix} 0.1 & 0.7 \\ -0.3 & 0.4 \end{bmatrix} \cdot \begin{bmatrix} -0.2 & 0.8 \\ 0.5 & -0.6 \end{bmatrix} = \begin{bmatrix} 0.33 & -0.34 \\ 0.26 & -0.48 \end{bmatrix}$$

$$\mathbf{G} \cdot \mathbf{F} = \begin{bmatrix} -0.2 & 0.8 \\ 0.5 & -0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.1 & 0.7 \\ -0.3 & 0.4 \end{bmatrix} = \begin{bmatrix} -0.26 & 0.18 \\ 0.23 & 0.11 \end{bmatrix}$$

$\mathbf{F} \cdot \mathbf{G} \neq \mathbf{G} \cdot \mathbf{F} \implies$  Matrix multiplication is not commutative.

**Problem 4:**

Textbook: 2-10

$$\text{a. } \mathbf{C} = \mathbf{A} \cdot \mathbf{B} = \begin{bmatrix} 0.1 & -0.3 & 0.2 \\ 0.6 & -0.1 & 0.4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix} = \begin{bmatrix} 1.7 \\ 1.8 \end{bmatrix}$$

$$\text{b. } \mathbf{C} = \mathbf{A} \cdot \mathbf{B} = \begin{bmatrix} 0.3 \\ 1.2 \\ -0.5 \end{bmatrix} \cdot \begin{bmatrix} -0.2 & 0.7 & 0.5 \end{bmatrix} = \begin{bmatrix} -0.06 & 0.21 & 0.15 \\ -0.24 & 0.84 & 0.6 \\ 0.10 & -0.35 & -0.25 \end{bmatrix}$$

$$c. \quad C = A \cdot B = \begin{bmatrix} -0.2 & 0.7 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 0.3 \\ 1.2 \\ -0.5 \end{bmatrix} = [0.53]$$

$$d. \quad C = A \cdot B = \begin{bmatrix} 0.4 & 0.6 \\ 1.3 & -0.2 \\ -0.8 & 1.5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & -1 & 4 \\ 6 & -4 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 4.4 & -1.2 & -0.4 & 4.6 \\ 1.4 & 4.7 & -1.3 & 4.2 \\ 7.4 & -8.4 & 0.8 & 4.3 \end{bmatrix}$$

**Problem 5:**

Textbook: 2-12

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 & 2 \\ 1 & 4 & -3 \\ 2 & 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{rcl} 3c_{11} + c_{12} + 2c_{13} & & = 1 \\ -c_{11} + 4c_{12} + 5c_{13} & & = 0 \\ 2c_{11} - 3c_{12} + 3c_{13} & & = 0 \\ & 3c_{21} + c_{22} + 2c_{23} & = 0 \\ & -c_{21} + 4c_{22} + 5c_{23} & = 1 \\ & 2c_{21} - 3c_{22} + 3c_{23} & = 0 \end{array}$$

$$\begin{array}{rcl} & 3c_{31} + c_{32} + 2c_{33} & = 0 \\ & -c_{31} + 4c_{32} + 5c_{33} & = 0 \\ & 2c_{31} - 3c_{32} + 3c_{33} & = 1 \end{array}$$

Solving the above equations yields the inverse =  $\begin{bmatrix} 0.321 & 0.155 & -0.060 \\ -0.107 & 0.060 & 0.131 \\ -0.036 & -0.202 & 0.155 \end{bmatrix}$

**Problem 6:**

Textbook: 2-16

$$\text{tr}(B) = -2 + (-3) + (-2) = -7$$

**Problem 7:**

Textbook: 2-20

$$(a) \quad V_1 V_2^T = \begin{bmatrix} 2 & 6 & -3 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} = [-6]$$

$$(b) \quad V_1 V_2^T = \begin{bmatrix} 0.6 \\ 0.2 \\ 0.0 \\ 0.3 \end{bmatrix} \cdot \begin{bmatrix} -0.4 & 0.1 & -0.5 & 0.2 \end{bmatrix} = \begin{bmatrix} -0.24 & 0.06 & -0.30 & 0.12 \\ -0.08 & 0.02 & -0.10 & 0.04 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ -0.12 & 0.03 & -0.15 & 0.06 \end{bmatrix}$$