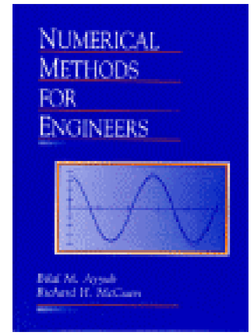
University of Maryland at College Park

Department of Civil and Environmental Engineering

ENCE 203 – Computation Methods in Civil Engineering II



EXAM II SOLUTION

(Closed Book & Notes, 8.5''x11" sheet is permitted)

Wednesday, May 2, 2001

10:00 am – 10:50 am, EGR 3106

Instructor: Dr. I. Assakkaf

"***Show your work & state all your assumptions***"

Student Name:___________SAMPLE________________

SSN:___123-45-6789_

Grade:___100 ☺___

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Problem 1 (25 points)

For the following set of linear simultaneous equations, use Gauss-Seidel iteration to solve the equations with initial values of $x_1 = x_2 = 0$ and $x_3 = 1$:

$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$
$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$

Use two iterations with 4 significant figures. How would you improve the accuracy of the method?

*** SOLUTION ***

To make the equations diagonally dominant, and to avoid divergence problems with the method, these equations should be rearrange as follows:

$$\begin{matrix} 0.1x_1 + 7x_2 - 0.3x_3 = -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 = 71.4 \\ 3x_1 - 0.1x_2 - 0.2x_3 = 7.85 \end{matrix} \quad \Longrightarrow \quad \begin{matrix} 3x_1 - 0.1x_2 - 0.2x_3 = 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 = -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 = 71.4 \end{matrix}$$

Therefore,

$$x_1 = \frac{7.85 + 0.1x_2 + 0.2x_3}{3}, \quad x_2 = \frac{-19.3 - 0.1x_1 + 0.3x_3}{7}, \text{ and } \quad x_3 = \frac{71.4 - 0.3x_1 + 0.2x_2}{10}$$

Iteration 1, $i = 1$:

$$x_1 = \frac{7.85 + 0.1(0) + 0.2(1)}{3} = 2.683$$

$$x_2 = \frac{-19.3 - 0.1(2.683) + 0.3(1)}{7} = -2.753$$

$$x_3 = \frac{71.4 - 0.3(2.683) + 0.2(-2.753)}{10} = 7.004$$

Iteration 2, $i = 2$:

$$x_1 = \frac{7.85 + 0.1(-2.753) + 0.2(7.004)}{3} = 2.992$$

$$x_2 = \frac{-19.3 - 0.1(2.992) + 0.3(7.004)}{7} = -2.500$$

$$x_3 = \frac{71.4 - 0.3(2.992) + 0.2(-2.500)}{10} = 7.000$$

The accuracy can be improved by using more iteration cycles and more significant figures.

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Problem 2 (30 points)

The following set of equations is given in matrix form as

$$\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

(a) Identify the matrix of coefficients, the vector of unknowns, and the vector of constants.

(b) Perform LU decomposition on the matrix of coefficients $\boldsymbol{A}$ (i.e., Find the lower and upper triangular matrices $\boldsymbol{L}$ and $\boldsymbol{U}$ such that $\boldsymbol{LU} = \boldsymbol{A}$).

(c) Solve this system of equations using the method of determinants.

*** SOLUTION ***

(a) Identify $\boldsymbol{A}$, $\boldsymbol{X}$ and $\boldsymbol{C}$:

$$A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}, \qquad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

(b) $l_{i1} = a_{i1}$ for $i = 1,2$ $\qquad u_{1j} = \dfrac{a_{1j}}{l_{11}}$ for $j = 2$

$$l_{11} = a_{11} = 1$$
$$l_{21} = a_{21} = 4$$

$$u_{12} = \frac{a_{12}}{l_{11}} = \frac{1}{1} = 1$$

$$l_{nn} = a_{nn} - \sum_{k=1}^{n-1} l_{nk} u_{kn}$$

$$l_{22} = a_{22} - \sum_{k=1}^{2-1} l_{2k} u_{k2} = -2 - l_{21}u_{12} = 2 - (4)(1) = -6$$

Therefore,

$$L = \begin{bmatrix} 1 & 0 \\ 4 & -6 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(c) Solution using the Method of Determinants:

$$|A| = \begin{vmatrix} 1 & 1 \\ 4 & -2 \end{vmatrix} = -2(1) - (1)(4) = -6$$

$$x_1 = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix}}{-6} = \frac{2(-2) - (1)(3)}{-6} = \frac{-7}{-6} = \underline{1.167}$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}}{-6} = \frac{3(1) - (2)(4)}{-6} = \frac{-5}{-6} = \underline{0.8333}$$

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Problem 3 (25 points)
For the following set of data:

x	0	1	2	3	4
$f(x)$	0	0.5	0.75	0.79	0.99

(a) Construct a finite-difference table and numerically evaluate the first, second, and third derivative at $x = 1$ using forward differences.

(b) Use the Simpson's 1/3-Rule to numerically evaluate the integral $\int_0^4 f(x)dx$. How can you reduce the error in your estimate of the integral?

*** SOLUTION ***

(a) Finite-difference Table:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	0				
		0.5			
1	0.5		-0.25		
		0.25		0.04	
2	0.75		-0.21		0.33
		0.04		0.37	
3	0.79		0.16		
		0.20			
4	0.99				

From the table,

$$\frac{df(1)}{dx} \approx \frac{0.25}{2-1} = 0.25 \quad , \quad \frac{d^2 f(1)}{dx^2} \approx \frac{-0.21}{(2-1)^2} = -0.21, \text{ and}$$

$$\frac{d^3 f(1)}{dx^3} \approx \frac{-0.37}{(2-1)^3} = -0.37, \text{ and}$$

(b) Simpson's 1/3-Rule:

$$\int_{x_1}^{x_n} f(x)dx \approx \sum_{i=1,3,5}^{n-2} \frac{x_{i+1} - x_i}{3}\left[f(x_i) + 4f(x_{i+1}) + f(x_{i+2})\right]$$

$$\int_0^4 f(x)dx \approx \sum_{i=1,3}^{5-2} \frac{1}{3}\left[f(x_i) + 4f(x_{i+1}) + f(x_{i+2})\right]$$

$$= \frac{1}{3}\left[f(x_1) + 4f(x_2) + f(x_3) + f(x_3) + 4f(x_4) + f(x_5)\right]$$

$$= \frac{1}{3}\left[0 + 4(0.5) + 0.75 + 0.75 + 4(0.79) + 0.99\right] = \underline{2.550}$$

The accuracy can be improved by
1. Smaller step size (does not apply in this case),or
2. Application of Simpson's 3/8-rule for the first 4 points, and then the Trapezoidal rule for the last pair of points.

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Problem 4(20 points)
Based on the following set of data points, the functional value, $f(1.35)$, resulting from using Newton's finite-difference interpolating polynomial is 3.57:

i	1	2	3
x	0.5	1.0	1.5
$f(x)$	2.2	f_2	3.9

What is the value of f_2 that is missing in the table?

*** SOLUTION ***

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
0.5	2.2		
		$f_2 - 2.2$	
1.0	f_2		$6.1-2f_2$
		$3.9 - f_2$	
1.5	3.9		

Newton's Method:

$$f(x) = f(x_0) + n[\Delta f(x_0)] + \frac{n(n-1)}{2!}\Delta^2 f(x_0)$$

$$n = \frac{x - x_0}{\Delta x} = \frac{1.35 - 0.5}{0.5} = 1.7$$

$$f(1.35) = 2.2 + 1.7(f_2 - 2.2) + \frac{1.7(1.7-1)}{2}(6.1 - 2f_2)$$

$$2.2 + 1.7f_2 - 3.74 + 0.595(6.1 - 2f_2) = f(1.35) = 3.57$$

$$2.2 + 1.7f_2 - 3.74 + 3.6295 - 1.19f_2 = 3.57$$

$$0.51f_2 = 1.4805$$

or

$$f_2 = \frac{1.4805}{0.51} = 2.9$$

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