INTRODUCTION TO ENGINEERING ECONOMICS

• A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

by

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ENCE 202
Spring 2000
Department of Civil and Environmental Engineering
University of Maryland

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Handout 9

Economic Analysis of Alternatives

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- Net Cash Flow of Investment Opportunities
  - Payments and disbursements need to be determined. Then a net cash flow can be developed

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Economic Analysis of Alternatives

- Present-Worth Amount
  - It is the difference between the equivalent receipts and disbursements at the present.
  - Assume F_t is a cash flow at time t, the present worth (PW) is

$$PW(i) = \sum_{t=0}^{n} F_t(P/F,i,t) = \sum_{t=0}^{n} F_t(1+i)^{-t}$$

for any interest -1< i <∞.

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Economic Analysis of Alternatives

- Annual Equivalent Amount
  - The annual equivalent amount is the annual equivalent receipts minus the annual equivalent disbursements of a cash flow. It is used for repeated cash flows per year.

$$AE(i) = PW(i)(A/P,i,n) = \left[\sum_{t=0}^{n} F_t(1+i)^{-t}\right]\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

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Example 1

Given the following cash flow:

Year end	Receipts	Disbursements
0	0	-1000
1	400	0
2	900	-1000
3	400	-
4	900	-1000
.	.	.
n-2	900	-1000
n-1	400	0
n	900	0

Therefore, AE(10) = [-1000+400(P/F,10,1)+900(P/F,10,2)](A/P,10,2)

or AE(10) = [-1000+400(0.9091)+900(0.8265)](0.5762) = 61.9

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Discounted Present Worth Analysis

- Often in engineering economic studies, as well as in general financial analyses, a discounted present worth analysis is made of each alternative under consideration.
- It involves calculating the *equivalent* present worth or present value of all the dollar amounts involved in the alternative to determine its present worth.

Definition:

The present worth is *discounted* at a predetermined rate of interest called the *minimum attractive rate of return* (MARR or i^*).

The MARR is usually equal to the current rate of interest for borrowed capital plus an additional rate for such factors as risk, uncertainty, and contingencies.

$$\text{MARR} = i^* = i + i(\text{risk})$$

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Example 2

The Ace-in-the-Hole Construction Company is considering three methods of acquiring company pickups for use by field engineers. The alternatives are:

A. Purchase the pickups for $7,200 each and sell after 4 years for an estimated $1,200 each.

B. Lease the pickups for 4 years for $2,250 per year paid in advance at the beginning of each year. The contractor pays all operating and maintenance costs on the pickups and the leasing company retains ownership.

C. Purchase the pickups on special time payments with $750 down now and $2,700 per year at the end of each year for 3 years. Assume the pickups will be sold after 4 years for $1,200 each.

If the contractor's MARR is 15%, which alternative should he choose?

Note: All alternatives involve equal lives.

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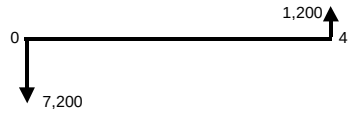
Example 2 (continued)

To solve, calculate the net present worth (NPW) of each alternative at 15% and select the least costly alternative:

A

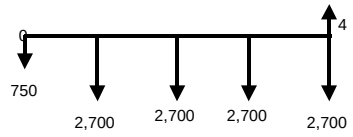


$NPW_A = -7,200 + 1,200(P/F),15,4) = \underline{-\$6,514}$

B

$NPW_B = -2,250 - 2,250(P/A),15,3) = \underline{-\$7,387}$

C



$NPW_C = -750 - 2,700(P/A),15,4)+1,200(P/A),15,4) = \underline{-\$7,772}$

The least costly alternative is A

Example 3

The engineering estimate for the cost of installing a new Astro-Turf for the college football field is $378,000 and is guaranteed for 5 years. Another artificial turf, the MACHO-TURF, is advertised for $494,000, installed, and guaranteed to last 8 years. Neglecting salvage value, which turf should be selected:

(a) if the MARR is 12%?

(b) if the MARR is 20%?

Example 3 (continued)

(a)

MARR = 12%

NAC_{AT} = 378,000 (*A*/*P*,12,5) = $104,861 / yr

NAC_{MT} = 494,000 (*A*/*P*,12,8) = $99,444 / yr

Choose Macho-Turf (Lower cost)

(b)

MARR = 20%

NAC_{AT} = 378,000 (*A*/*P*,20,5) = $126,396 / yr

NAC_{MT} = 494,000 (*A*/*P*,20,8) = $128,741 / yr

Choose Astro-Turf (Lower cost)

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What to do When Alternatives Involve Different Lives

Approach 1.

Truncate (cut off) the longer-lived alternative(s) to equal the shorter lived alternative and assume a salvage value for the unused portion of the longer lived alternatives. Then make the comparison on the basis of equal lives.

Approach 2.

Assume equal replacement conditions (costs and incomes) for each alternative and compute the discounted present worth on the basis of the least common multiple of lives for all alternatives.

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Example 4

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A contractor is considering the purchase of either a new track-type tractor for $73,570, which has a 6-year life with an estimated net annual income of $26,000 and a salvage value of $8,000, or a used track-type tractor for $24,680, with an estimated life of 3 years and no salvage value and an estimated net annual income of $12,000.

If the contractor's MARR is 20%, which tractor, if any, should she choose?

Example 4 (continued)

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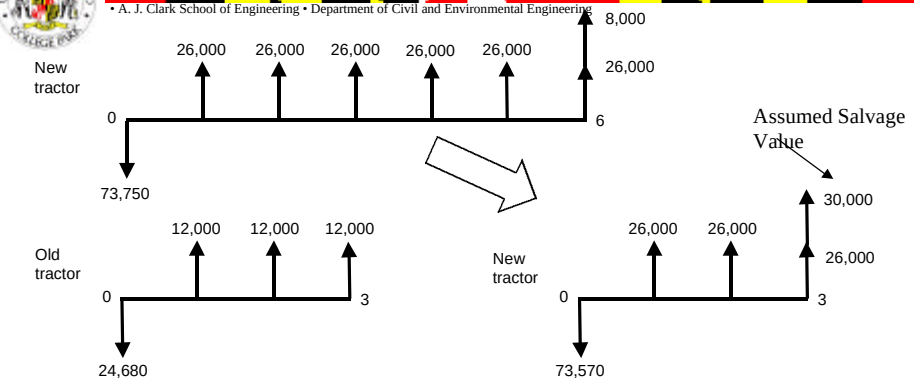
Approach 1. ***(comparison on the basis of equal lives)***



NPW_{new} = -73,570 + 26,000(P/**A**,20,3) + 30,000(P/**F**,20,3) = -73,570 + 26,000(2.10648) + 30,000(0.5787) = -$1,443

NPW_{old} = -24,680 + 12,000(P/A,20,3) = -24,680 + 12,000(2.10648) = + $597

Conclusion: Old tractor is a better Alternative

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Example 4 (continued)

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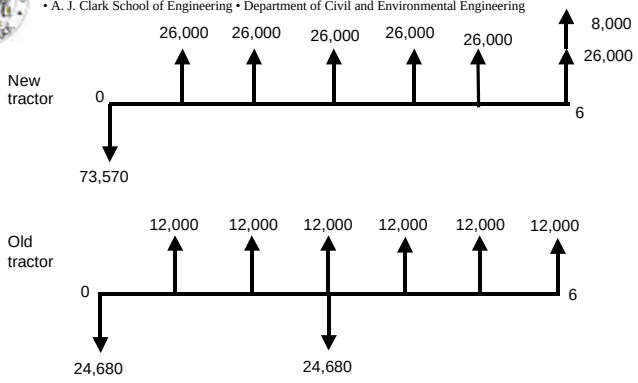
Approach 2. ***(comparison on the basis of equal replacement conditions)***



NPW_{new} = -73,570 + 26,000(P/**A**,20,6) + 8,000(P/**F**,20,6) = -73570 + 26,000(3.32551) + 8,000(0.33490) = $15,570

NPW_{old} = -24,680 + 12,000(P/**A**,20,6) - 24,680(P/**F**,20,3) = -24,680 + 12,000 (3.32551) - 24,680(0.57870) = +$944

Conclusion: New tractor is a better Alternative

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Rate of Return (ROR) Analysis

- Knowing the anticipated *rate of return* of an investment permits decision maker to have more "perceived" confidence in its decision!

Definition:

*The **rate of return*** of a proposed investment is that interest rate which makes the discounted present worth of the investment equal to zero.

- To calculate the *rate of return,* simply set up the equation to be equal to zero and solve for *i*.

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Example 5

A contractor is considering the purchase of either a new track-type tractor for $73,570, which has a 6-year life with an estimated net annual income of $26,000, or a used track-type tractor for $24,680, with an estimated life of 3 years and no salvage value and an estimated net annual income of $12,000.

If the contractor's MARR is 20%, which tractor, if any, should be chosen?

Example 5 (continued)

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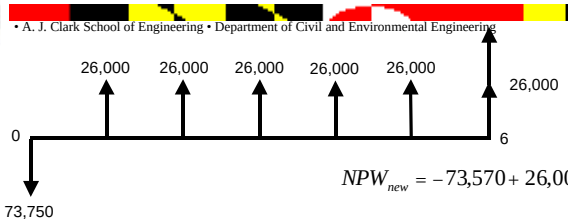
Approach 1. ***(comparison on the basis of equal lives)***

New tractor

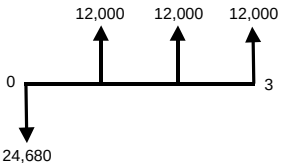


$$NPW_{new} = -73{,}570 + 26{,}000\left(\frac{P}{A}, i, 6\right) = 0$$

$$\left(\frac{P}{A}, i, 6\right) = \frac{73{,}570}{26{,}000} = 2.82962 = \frac{(1+i)^6 - 1}{i(1+i)^6}$$

$$i_{new} = 26.9\%$$

Old tractor



$$NPW_{old} = -24{,}680 + 12{,}000\left(\frac{P}{A}, i, 3\right) = 0$$

$$\left(\frac{P}{A}, i, 3\right) = \frac{24{,}680}{12{,}000} = 2.05667 = \frac{(1+i)^3 - 1}{i(1+i)^3}$$

$$i_{old} = 21.5\%$$

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Example 5 (continued)

Iterative Solution

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$$NPW_{new} = -73{,}570 + 26{,}000\left(\frac{P}{A}, i, 6\right) = 0$$

$$\left(\frac{P}{A}, i, 6\right) = \frac{73{,}570}{26{,}000} = 2.82962 = \frac{(1+i)^6 - 1}{i(1+i)^6} \Rightarrow \boxed{i_{new} = \frac{(1+i)^6 - 1}{2.82962(1+i)^6}}$$

$$NPW_{new} = -24{,}680 + 12{,}000\left(\frac{P}{A}, i, 3\right) = 0$$

$$\left(\frac{P}{A}, i, 6\right) = \frac{24{,}680}{12{,}000} = 2.05667 = \frac{(1+i)^3 - 1}{i(1+i)^3} \Rightarrow \boxed{i_{old} = \frac{(1+i)^3 - 1}{2.05667(1+i)^3}}$$

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Example 5 (continued)

Iterative Solution

$$i_{new} = \frac{(1+i)^6 - 1}{2.82962(1+i)^6}$$

i	NPW_{new}
0.200	12893
0.235	5877
0.254	2498
0.262	1027
0.266	416
0.268	168
0.268	67
0.268	27
0.269	11
0.269	4
0.269	2
0.269	1
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0

$i_{new} = 26.9\%$

$$i_{old} = \frac{(1+i)^3 - 1}{2.05667(1+i)^3}$$

i	$NPWold$
0.150	2719
0.167	1985
0.180	1415
0.190	990
0.198	683
0.203	466
0.207	316
0.210	214
0.212	144
0.213	97
0.214	65
0.214	43
0.215	29
0.215	19
0.215	13
0.215	9
0.215	6
0.215	4
0.215	3
0.215	2
0.215	1
0.215	1
0.215	1
0.215	0
0.215	0

$i_{new} = 21.5\%$

Example 5 (continued)

- If MARR is 20%
- Then, the new tractor is selected because i = 26.9 % is greater than MARR = 20%

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Rate of Return (ROR) Analysis

- If we assume the salvage value for the new tractor to be $30,000 after 3 years, the NPW_{new} will be:

  $NPW_{old} = -24,680 + 12,000(P/A,i,3) = 0$

  i_{old} = 21.5%

  $NPW_{new} = -73,570 + 26,000(P/A,i,3) + 30,000(P/F,i,3) = 0$

  i_{new} = 18.9%

  Before the decision can be reached
  YOU MUST KNOW YOUR MARR.

- If MARR = 20% and 3 year analysis period, we choose old tractor.
- If MARR = 30%, we choose neither tractor - do nothing alternative.
- If the MARR was 15%, which alternative should we select then?

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Rate of Return (ROR) Analysis

- Both NPW_{old} and NPW_{new} *exceed* the MARR = 15%.
- But since the old tractor yields a higher MARR, should it not be selected?
- To answer this question, determine each alternative's net present worth at 15%.

  $NPW_{old} = -24,680 + 12,000(P/A,15,3) = \$2,719$

  $NPW_{new} = -73,570 + 26,000(P/\mathbf{A},15,3) + 30,000(P/\mathbf{F},15,3) = \$5,519$

  According to the above NPW analysis, the new tractor yields a higher value for a MARR of 15%?

  Shouldn't the alternative with the higher rate of return would yield the higher NPW regardless of the assumed interest rate?

  NO IT SHOULD NOT!

  The initial investments in the tractor examples we used are *not the same*.

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Incremental Rate of Return (ROR) Analysis

- When we examined the rate of return of each alternative, we have ignored their respective differences in initial cash flows. Therefore, we can obtain misleading results through such an analysis.
- To deal with the problem of unequal initial investments, *an incremental rate of return (IROR) analysis* is required.
-
- "For alternatives that have a satisfactory rate of return (ROR), what is the IROR of the difference in the cash flows of the alternatives?"
- To make this analysis, first arrange the alternatives in ascending order of initial cash flow. Then compare alternatives, two by two, alternatively rejecting the alternative with the lower IROR.

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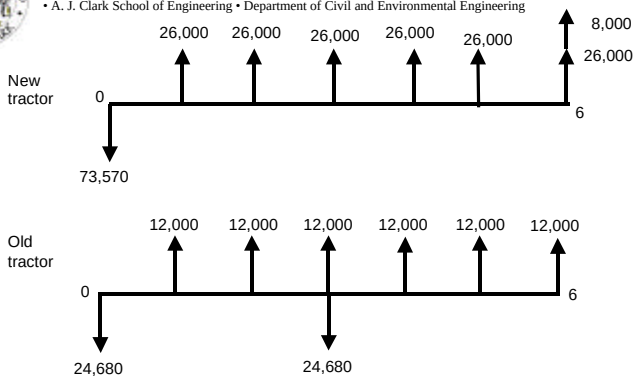
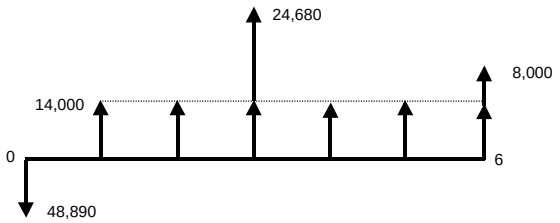
Incremental Rate of Return (ROR) Analysis



Incremental Rate of Return (ROR) Analysis

$NPW_{new-old} = -48{,}890 + 14{,}000(P/A,i,6) + 24{,}680(P/F,i,3) + 8{,}000(P/F,i,6) = 0$

$i = 30.9\%$



Incremental Rate of Return (ROR) Analysis

- While the initial investment of $24,680 for the old tractor will yield a ROR of 21.5%, the incremental increase in initial investment of $48,890 (by purchasing the new tractor) will yield an IROR of 30.9%.
- Now that all the rates of return are known, a decision can be reached *which is dependent on the MARR.*
  - For a MARR of 20% the ROR of the new tractor is too low, and therefore the old tractor is chosen.
  - For a MARR of 15% both alternatives exceed it and we have to examine the IROR.
  - In this case the IROR is higher than the MARR, so we should choose the new tractor.