INTRODUCTION TO ENGINEERING ECONOMICS

• A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

by

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ENCE 202
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Department of Civil and Environmental Engineering
University of Maryland

The Time Value of Money (TVM)

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- Money has a time value
- One dollar today is worth more than $1 tomorrow
- Failure to pay the bills results in additional charge termed

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The Interest (i)

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- Interest is usually expressed as a percentage of the amount owed.
- It is due and payable at the close of each period of time involved in the agreed transaction (usually every month).

Example:

If $ 1,000.00 is borrowed at 14% interest, then interest on the ***principal of*** $ 1,000.00 after one year is 0. 14 x 1, 000, or $140.00.

If the borrower pays back the total amount owed after one year, she will pay $1,140.00.

If she does not pay back any of the amount owed after one year, then normally the interest owed, but not paid, is considered now to be additional principal, and thus the interest is ***compounded.***

*A*fter two years she will owe $1,140.00+0.14 X 1,140.00,or 1,299.60.

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Equivalency

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The banker normally does not care whether you pay him $1,140.00 after one year or $1,299.60 after two years. To him, the three values ($1,000, $1,140, and $1,299.60) are ***equivalent****.*

$ 1,000 today is equivalent to $1,140 one year from today,

$ 1,000 today is equivalent to $1,299.60 two years from today.

The three values are not equal but *equivalent.*

Note:

1.***The concept of equivalence involves time and a specified rate of interest****. The* three preceding values are only equivalent for an interest rate of 14%, and then only at the specified times.

2. *Equivalence* means that one sum or series differs from another only by the accumulated interest at rate ***i*** for ***n*** periods of time.

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Financial Engineering Analysis

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- Single payment
- Uniform series of payments
- Discounted present worth analysis
- Rate of return analysis

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Cash Flow

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- Cash flow over time: Upward arrow means positive flow, downward arrow means negative flow. There are two cash flows to each problem (borrower and lender flows).
- Net cash flow: The arithmetic sum of receipts (+) and disbursements (-) that occur at the same point in time.

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Cash Flow

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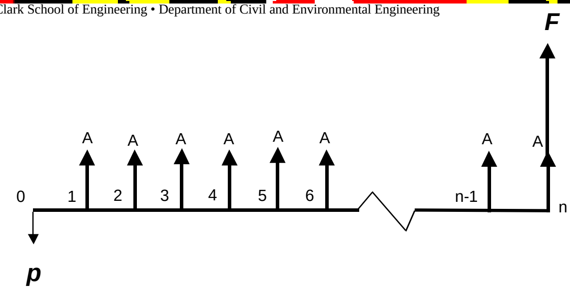

Notations

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P = a present single amount of money

F = a future single amount of money, after n periods of time

A = a series of n equal payments

i = the rate of interest per interest period (usually one year)

n = the number of periods of time (usually years)

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Single Payment

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- Single Payment Compound-Amount Factor (SPCAF)

$$F = P(1+i)^n$$

OR

$$F = \left(\frac{F}{P}, i, n\right)$$

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Single Payment

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- Single Payment Present-Worth Factor (SPPWF)

$$P = \frac{F}{(1+i)^n}$$

OR

$$P = \left(\frac{P}{F}, i, n\right)$$

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Single Payment Analysis

To calculate the future value F of a single payment P after n periods at an interest rate i, we make the following calculation:

At the end of the first period: $F_1 = P + Pi$
At the end of the second period: $F_2 = P + Pi + (P + Pi)i = P(1 + i)^2$
At the end of the nth period: $F_n = P(1 + i)^n$

The future single amount of a present single amount is

$$F = P(1 + i)^n$$

Single Payment Analysis

Note:

F is related to *P* by a factor which depends only on *i* and *n*. This factor, termed *the single payment compound amount factor* (SPCAF), makes *F equivalent to P.*

SPCAF may be expressed in a functional form:

$$(1+i)^n = \left(\frac{F}{P}, i, n\right) \quad \text{or} \quad F = P\left(\frac{F}{P}, i, n\right)$$

The present single amount of a future single amount is

$$P = \frac{F}{(1+i)^n} \quad \text{or} \quad P = F\left(\frac{P}{F}, i, n\right)$$

Single Payment Analysis

Note:

The factor $1/(1+i)^n$ is called the *present worth compound amount factor* (PWCAF)

$$\frac{1}{(1+i)^n} = \left(\frac{P}{F}, i, n\right)$$

Example 1: Single Payment

A contractor wishes to set up a revolving line of credit at the bank to handle her cash flow during the construction of a project.
She believes that she needs to borrow \$12,000 with which to set up the account, and that she can obtain the money at 1.45% per month.

If she pays back the loan and accumulated interest after 8 months, how much will she have to pay back?

$F = 12{,}000(1 + 0.0145)^8 = 12{,}000(1.122061) = 13{,}464.73 = \underline{\$13{,}465}$

The amount of interest will be:

\$13,465 - 12,000 = \$1,465.

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Example 2: Single Payment

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A construction company wants to set aside enough money today in an interest-bearing account in order to have $ 100,000 five years from now for the purchase of a replacement piece of equipment.

If the company can receive 8% interest on its investment, how much should be set aside now to collect the $100,000 five years from now?

P = 100,000/(I + 0.08)^5 =100,000/(1.46933) = $68,058.32 = <u>*$68,060*</u>

To solve this problem you can also use the interest tables.

P =100,000 (P/F, 8,5) = 100,000(0.6805832) $68,058.32 = <u>*$68,060*</u>

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Uniform Series of Payments Analysis

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- Uniform (Equal payment) Series Compound-Amount Factor (USCAF)

$$F = A\left(\frac{(1+i)^n - 1}{i}\right)$$

OR

$$F = \left(\frac{F}{A}, i, n\right)$$

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Uniform Series of Payments Analysis

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- Uniform (Equal payment) Series Sinking-Fund Factor (USSFF)

$$A = F\left(\frac{i}{(1+i)^n - 1}\right)$$

OR

$$A = \left(\frac{A}{F}, i, n\right)$$

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Interest Formulas

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- Uniform (Equal payment) Series Capital-Recovery Factor (USCRF)

$$A = P\left(\frac{i(1+i)^n}{(1+i)^n - 1}\right)$$

OR

$$A = \left(\frac{A}{P}, i, n\right)$$

NOTE: This is the case of loans (mortgages)

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Interest Formulas

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- Uniform (Equal payment) Series Present-Worth Factor (USPWF)

$$P = A\left(\frac{(1+i)^n - 1}{i(1+i)^n}\right)$$

OR

$$P = \left(\frac{P}{A}, i, n\right)$$

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Uniform Series of Payments Analysis

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- Often payments or receipts occur at regular intervals, and such uniform values can be handled by the use of additional functions. Another symbol:

  A = uniform ***end-of-period*** payments or receipts continuing for a duration of ***n*** periods

- If a uniform amount A is invested at the end of each period for n periods at a rate of interest i per period, then the total equivalent amount F at the end of the n periods will be:

$$F = A\left[(1+i)^{n-1} + (1+i)^{n-2} + ... + (1+i) + 1\right]$$

By multiplying both sides of above equation by (1+i) and subtracting from the original equation, the following expression is obtained:

$$Fi = A(1+i)^n - 1$$

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Which can be rearrange to give

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

The relationship can also be expressed in a functional form as

$$F = A\left(\frac{F}{A}, i, n\right)$$

$[(1+i)^n - 1]/i$ is called the *uniform series compound amount factor* (USCAF)

It can also be shown that

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

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Which can be rearrange to give

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

The relationship can also be expressed in a functional form as

$$F = A\left(\frac{F}{A}, i, n\right)$$

$[(1+i)^n - 1]/i$ is called the *uniform series compound amount factor* (USCAF)

It can also be shown that

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

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Which can be expressed in a functional form as

$$A = F\left(\frac{A}{F}, i, n\right)$$

The relationship *i* / [(1+*i*)^n - 1]is termed as the uniform series sinking fund factor (USSFF)

Recall that $F = P(1+i)^n$

Hence

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \quad \text{or} \quad P = A\left(\frac{P}{A}, i, n\right)$$

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Uniform Series of Payments Analysis

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The relationship $\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ is called the *uniform series present worth factor* (USPWF)

Also

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \quad \text{or} \quad A = P\left(\frac{A}{P}, i, n\right)$$

The relationship $\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$ is called the uniform series capital recovery factor (USCRF)

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Cash Flow Diagram for Single Payment

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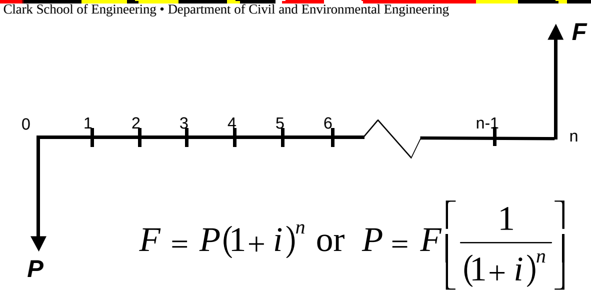

$$F = P(1+i)^n \text{ or } P = F\left[\frac{1}{(1+i)^n}\right]$$

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Cash Flow Diagram for Single Payment

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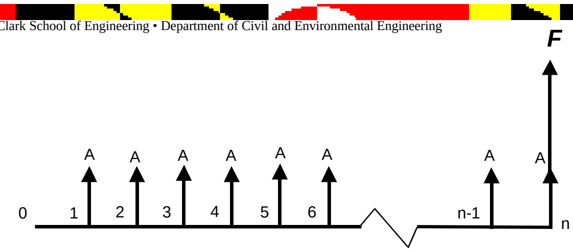

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] \text{ or } A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

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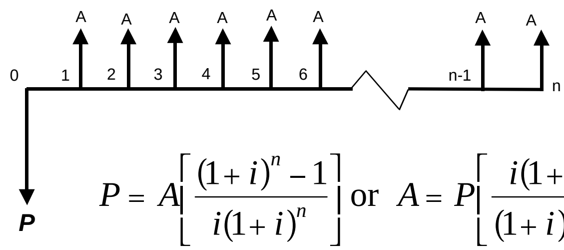
Cash Flow Diagram for Single Payment



$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \text{ or } A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

Example 3

A piece of construction equipment costs $45,000 to purchase. Fuel, oil, grease, and minor maintenance are estimated to cost $12.34 for each hour that the equipment is used. The tires cost $3,200 to replace (estimated to occur every 2,800 hours of use), and major repairs of $6,000 are expected after 4,200 hours of use.
The piece of equipment is expected to last for 8,400 hours, after which it will have an estimated salvage value of 10% of the purchase price.

How much should the owner of the equipment charge, per hour of use, if he expects to use the piece of equipment about 1,400 hours per year? Assume an annual interest rate of 15%.

Example 3 (continued)

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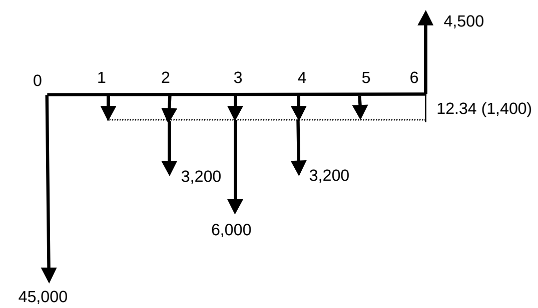
Cash Flow Diagram



n = 8,400/1,400 = 6 yrs,
n_T = 2800/1400 = 2 yrs,
n_R= 4200/1400 = 3yrs

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Example 3 (continued)

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n = 8,400/1,400 = 6 yrs, n_T = 2800/1400 = 2 yrs, n_R= 4200/1400 = 3yrs

A_1 = - 45,000(A/P,15,6) = - 45,000 (0.26424) = -11,890.80

A_2= - 12.34 (1,400) = -17,276.00

A_3=[- 3,200 (P/F,15,2) – 3,200 (P/F,15,4)] (A/P,15,6)
= [- 3,200 (0.75614) – 3,200 (0.57175)] (0.26424) = -1,122.78

A_4= - 6,000 (P/F,15,3)(A/P,15,6)
= - 6,000 (0.65752)(0.26424) = -1,042.46

A_5= + 4,500(A/F,15,6) = 4,500 (0.11424) = +514.08

A_T= the total annual cost = -30,817.96

The hourly cost = 30,817.96/1,400 = $22.01/hr

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \text{ or } A = P\left(\frac{A}{P}, i, n\right) \qquad A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \text{ or } P = A\left(\frac{P}{A}, i, n\right) \qquad \frac{1}{(1+i)^n} = \left(\frac{P}{F}, i, n\right)$$

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Example 3 (continued)

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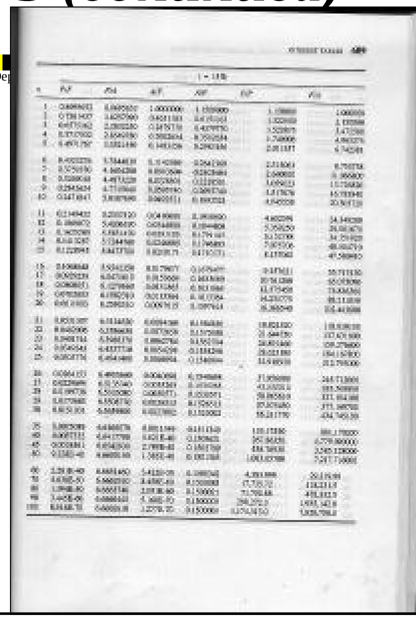

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