Duxbury Thomson Learning

Making Hard Decision

Third Edition

RISK ATTITUDES

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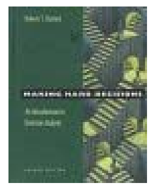

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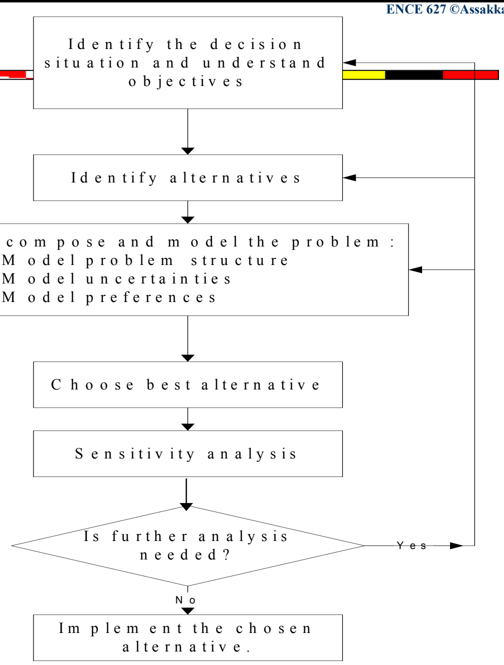
Modeling Decision Problems



The Decision-Analysis Flow Chart

[Source: R.Clemen, 1996]

Methodology for Modeling Decision

The Methodology of Modeling preferences is described in four chapters as follows:

Detailed Steps

- **<u>Risk Attitudes:</u> reviews fundamental risk attitude concepts.** — *Chapter 13*
- ***<u>The Following Chapters are not included in this course:</u>***
- **<u>Utility Axioms, Paradoxes, and Implications:</u> reviews utility problems and paradoxes.** — *Chapter 14*
- **<u>Conflicting Objectives I:</u> reviews fundamental objectiv and additive utility function.** — *Chapter 15*
- **<u>Conflicting Objectives II:</u> reviews Multiattribute utilit models with interactions** — *Chapter 16*

Modeling Preferences

- Why should we worry about modeling preferences?
- Because virtually every decision involves some kind of trade-off.

Modeling Preferences

- In decision making under uncertainty, the fundamental trade-off question often is, How much risk is a decision maker willing to assume? After all, expected monetary value is not everything! Often the alternative that has the greatest EMV also involves the greatest risk.

Modeling Preferences

- Chapters 13 and 14 look at the role of risk attitudes in decision making.
- In Chapter 13, basic concepts are presented, and you will learn how to model your own risk attitude. We will develop the concept of a utility function. Modeling your preferences by assessing your utility function is a subjective procedure much like assessing subjective probabilities

Modeling Preferences

- Because a utility function incorporates a decision maker's attitude toward risk, the decision maker may decide to choose the alternative that maximizes his or her expected utility rather than expected monetary value.

Modeling Preferences

- Chapter 14 discusses some of the foundations that underline the use of utility functions. The essential reason for choosing alternatives to maximize expected utility is that such behavior is consistent with some fundamental choice and behavior patterns that we call axioms.

Modeling Preferences

- The paradox is that, even though most of us agree that intuitively the axioms are reasonable, there are cases for all of us when our actual choices are not consistent with the axioms. In many situations these inconsistencies have little effect on a decision maker's choices. But occasionally they can cause trouble, and the will chapter discuss some of these difficulties and their implications.

Modeling Preferences

- Dealing with risk attitudes is an important aspect of decision making under uncertainty, but it is only part of the picture. As we discussed in Section I, many problems involve conflicting objectives.

Modeling Preferences

- Decision makers must balance many different aspects of the problem, and try to accomplish many things at once. Even a simple decision such as deciding where to go for dinner involves trade-offs: **How far are you willing to drive? How much should you spend? How badly do you want Chinese food?**

Modeling Preferences

- Chapters 15 and 16 deal with modeling preferences in situations in which the decision maker has multiple and conflicting objectives. In both chapters, one of the fundamental subjective assessments that the decision maker must make is how to trade off achievement in one dimension against achievement in another.

Modeling Preferences

- Chapter 15 presents a relatively straightforward approach that is easy and intuitive, extending the introductory approach presented in Chapters 3 and 4. Chapter 16 extends the discussion to include interaction among utility attributes.

Risk

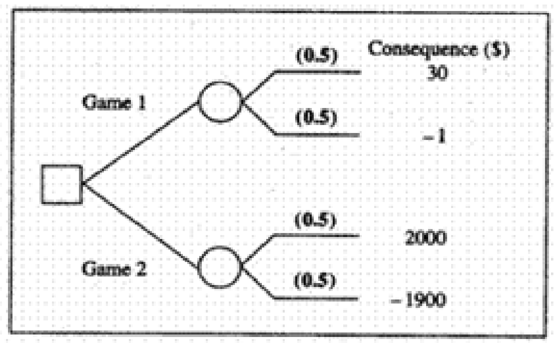
- **Basing decisions on expected monetary values (EMVs) is convenient, but it can lead to decisions that may not seem intuitively appealing.**
- **For example, consider the following two games. Imagine that you have the opportunity to play one game or the other, but only one time. Which one would you prefer to play? Your choice also is drawn in decision-tree form in Figure 13.1.**
- **Game 1: Win $30 with probability 0.5**

  Lose $1 with probability 0.5
- **Game 2: Win $2000 with probability 0.5**

  Lose $1900 with probability 0.5
- **Game 1 has an expected value of $14.50. Game 2, on the other hand, has an expected value of $50.00.**
- **If you were to make your choice on the basis of expected value, then you would choose Game 2. Most of us, however, would consider Game 2 to be riskier than Game 1, and it seems reasonable to suspect that most people actually would prefer Game 1.**

Risk

Figure 13.1
Two lottery games. Which game would you choose?



Risk

- **Using expected values to make decisions means that the decision maker is considering only the average or expected payoff. If we take a long-run frequency approach, the expected value is the average amount we would be likely to win over many plays of the game.**

- **But this ignores the range of possible values. After all, if we play each game 10 times, the worst we could do in Game 1 is to lose $10. On the other hand, the worst we could do in Game 2 is lose $19,000!**

- **Many of the examples and problems that we have considered so far have been analyzed in terms of expected monetary value (EMV). EMV, however, does not capture risk attitudes.**

Risk

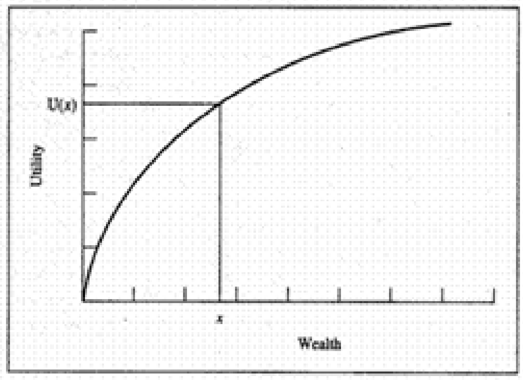
- Individuals who are afraid of risk or are sensitive to risk are called *risk-averse.*
- We could explain risk aversion if we think in terms of a *utility function* (Figure 13.2) that is curved and opening downward (the technical term for a curve with this shape is *concave).*
- This utility function represents away to translate dollars into "utility units." That is, if we take some dollar amount *(x),* we can locate that amount on the horizontal axis.
- Read up to the curve and then horizontally across to the vertical axis. From that point we can read off the utility value U*(x)* for the dollars we started with.

Risk

- A utility function might be specified in terms of a graph, as in Figure 13.2, or given as a table, as in Table 13.1. A third form is a mathematical expression. If graphed, for example, all of the following expressions would have the same general concave shape (opening downward) as the utility function graphed in Figure 13.2:
- $U(x) = \log(x)$
- $U(x) = 1 - e^{-x/R}$
- $U(x) = +\sqrt{x}$ [or $U(x) = x^{0.5}$]
- Of course, the utility and dollar values in Table 13.1 also could be converted into a table of values. The point is that the utility function makes the translation from dollars to utility regardless of its displayed form.

Risk

Figure 13.2
A utility function that displays risk aversion.



Risk

Table 13.1
A utility function in tabular form.

Wealth	Utility Value
2500	1.50
1500	1.24
1000	0.93
600	0.65
400	0.47
0	0.15

Risk Attitudes

■ We think of a typical utility curve as (1) upward sloping and (2) concave (the curve opens downward). An upward-sloping utility curve makes fine sense; it means that more wealth is better than less wealth, everything else being equal. Few people will argue with this. Concavity in a utility curve implies that an individual is risk-averse.

■Imagine that you are forced to play the following game:

Win $500 with probability 0.5

Lose $500 with probability 0.5

■Would you pay to get out of this situation? How much? The game has a zero expected value, so if you would pay something to get out, you are avoiding a risky situation with zero expected value.

■Generally, if you would trade a gamble for a sure amount that is less than the expected value of the gamble, you are risk-averse.

■Purchasing insurance is an example of risk-averse behavior. Insurance companies analyze a lot of data in order to understand the probability distributions associated with claims for different kinds of policies. Of course, this work is costly.

Risk Attitudes

■ To make up these costs and still have an expected profit, an insurance company must charge more for its insurance policy than the policy can be expected to produce in claims. Thus, unless you have some reason to believe that you are more likely than others in your risk group to make a claim, you probably are paying more in insurance premiums than the expected amount you would claim.

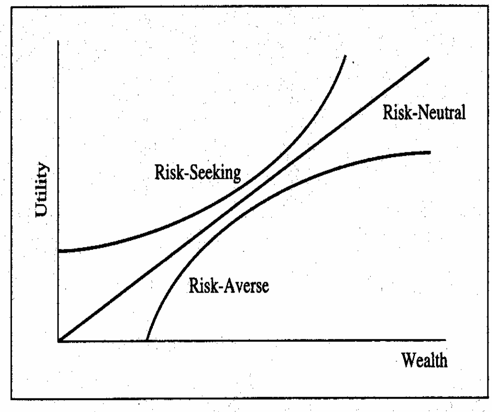
■ Not everyone displays risk-averse behavior all the time, and so utility curves need not be concave.

■ A convex (opening upward) utility curve indicates risk-seeking behavior (Figure 13.3).

■ The risk seeker might be eager to enter into a gamble; for example, he or she might pay to play the game just described. An individual who plays a state lottery exhibits risk-seeking behavior. State lottery tickets typically cost $1.00 and have an expected value of approximately 50 cents.

Risk Attitudes

Figure 13.3
Three different shapes for utility functions.



Risk Attitudes

- Finally, an individual can be risk-neutral.

- Risk neutrality is reflected by a utility curve that is simply a straight line. For this type of person, maximizing EMV is the same as maximizing expected utility. This makes sense; someone who is risk-neutral does not care about risk and can ignore risk aspects of the alternatives that he or she faces. Thus, EMV is a fine criterion for choosing among alternatives, because it also ignores risk.

- Although most of us are not risk-neutral, it often is reasonable for a decision maker to assume that his or her utility curve is nearly linear in the range of dollar amounts for a particular decision. This is especially true for large corporations that make decisions involving amounts which are small relative to their total assets.

Risk Attitudes

- In many cases, it may be worthwhile to use EMV in a first-cut analysis, and then check to see whether the decision would be sensitive to changes in risk attitude.
- If the decision turns out to be fairly sensitive (that is, if the decision would change for a slightly risk-averse or slightly risk-seeking person), then the decision maker may want to consider modeling his or her risk attitude carefully.
- If we have a utility function that translates from dollars to utility, how should we use it? The whole idea of a utility function is that it should help to choose from among alternatives that have uncertain payoffs. Instead of maximizing expected value, the decision maker should maximize expected utility.
- In a decision tree or influence-diagram payoff table, the net dollar payoffs would be replaced by the corresponding utility values and the analysis performed using those values. The best choice then should be the action with the highest expected utility.

Risk Tolerance and The Exponential Utility Function

- The assessment processworks well for assessing a utility function subjectively, and it can be used in any situation, although it can involve a fair number of assessments. An alternative approach is to base the assessment on a particular mathematical function, such as one of those that we introduced early in the chapter. In particular, let us consider the *exponential utility Junction:*

$$U(x) = 1 - e^{-x/R}$$

- This utility function is based on the constant e = 2.71828 ..., the base of natural logarithms. This function is concave and thus can be used to represent risk-averse preferences.
- As *x* becomes large, *U(x)* approaches 1. The utility of zero, U(0), is equal to 0, and the utility for negative *x* (being in debt) is negative.

Risk Tolerance and The Exponential Utility Function

- In the exponential utility function, *R* is a parameter that determines how risk- averse the utility function is.
- In particular, *R* is called the ***risk tolerance.*** Larger values of *R* make the exponential utility function flatter, while smaller values make it more concave or more risk-averse.
- Thus, if you are less risk-averse-if you can tolerate more risk—you would assess a larger value for *R* to obtain a flatter utility function. If you are less tolerant of risk, then you would assess a smaller *R* and have a more curved utility function.
- How can *R* be determined? A variety of ways exist, but it turns out that *R* has a very intuitive interpretation that makes its assessment relatively easy.

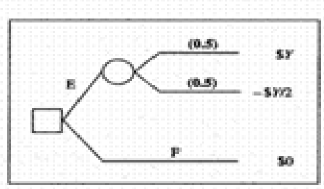
Risk Tolerance and The Exponential Utility Function

- Consider the gamble:

  Win $*Y* with probability 0.5

  Lose *$Y*/2 with probability 0.5

Figure 13.12 *Assessing your risk tolerance.* Find the largest value of *Y* for which you would prefer Alternative E.



- Would you be willing to take this gamble if *Y* were $100? $2000? $35,000? Or, framing it as an investment, how much would you be willing to risk ($*Y*/2) in order to have a 50% chance of tripling your money (winning *$Y* and keeping your $*Y*/2*)?* At what point would the risk become intolerable? The decision tree is shown in Figure 13.12.

Risk Tolerance and The Exponential Utility Function

- The largest value of *Y* for which you would prefer to take the gamble rather than not take it is approximately equal to your risk tolerance. This is the value that you can use for *R* in your exponential utility function.
- For example, suppose that after considering the decision tree in Figure 13.12 you conclude that the largest *Y* for which you would take the gamble is *Y* = \$900. Hence, *R* = \$900. Using this assessment in the exponential utility function would result in the utility function

$$U(x) = 1 - e^{-x/900}$$

- This exponential utility function provides the translation from dollars to utility units.
- Once you have your *R* value and your exponential utility function, it is fairly easy to find certainty equivalents.

Risk Tolerance and The Exponential Utility Function

- For example, suppose that you face the following gamble:

  Win \$2000 with probability 0.4

  Win \$1000 with probability 0.4

  Win \$500 with probability 0.2
- The expected utility for this gamble is
- EU = 0.4 U(\$2000) + 0.4 U(\$l000) + 0.2 U(\$500)

  = 0.4(0.8916) + 0.4(0.6708) + 0.2(0.4262) = 0.7102
- To find the CE we must work backward through the utility function. We want to find the value *x* such that U(x)= 0.7102. Set up the equation

$$0.7102 = 1 - e^{-x/900}$$

- Subtract 1 from each side to get

$$-0.2898 = -e^{-x/900}$$

Risk Tolerance and The Exponential Utility Function

- Multiply through to eliminate the minus signs:

$$0.2898 = e^{-x/900}$$

- Now we can take natural logs of both sides to eliminate the exponential term:

$$\ln(0.2898) = \ln(e^{-x/900})$$

- The rule (from algebra) is that $\ln(e^y) = y$.
- Now we simply solve for x:

$$\ln(0.2898) = x = -900[\ln(0.2898)] \quad = \$1114.71$$

- The procedure above requires that you use the exponential utility function to translate the dollar outcomes into utilities, find the expected utility, and finally convert to dollars to find the exact certainty equivalent. That can be a lot of work, especially if there are many outcomes to consider.
- Fortunately, an approximation is available from Pratt (1964) and also discussed in McNamee and Celona (1987).

Risk Tolerance and The Exponential Utility Function

- Suppose you can figure out the expected value and variance of the payoffs. Then the CE is approximately:
- Certainty Equivalent ≈ Expected Value - $\dfrac{0.5(Variance)}{RiskTolerance}$
- In symbols,

$$\text{CE} \approx \mu - \frac{0.5\sigma^2}{R}$$

- where μ and $\sigma 2$ are the expected value and variance, respectively. For example, in the gamble above, the expected value (EMV or μ) equals \$1300, and the standard deviation (σ) equals \$600. Thus, the approximation gives

$$\text{CE} \approx \$1300 - \frac{0.5(\$600)^2}{900} \approx \$\ 1100$$

- The approximation is within \$15. That's pretty good! This approximation is especially useful for continuous random variables or problems where the expected value and variance are relatively easy to estimate or assess compared to assessing the entire probability distribution.

Modeling Preferences Using Precision Tree

- It is straightforward to model a decision maker's risk preferences in Precision Tree. You need only choose the utility curve, and Precision Tree does the rest. It converts all end-node monetary values into utility values and calculates both expected utilities and certainty equivalents. You can choose one of the two built-in utility functions (the exponential or logarithmic) or you can define a customized utility curve.

- The logarithmic function is the easiest to use because it has no parameters, but it is also the least flexible because using it implies the same risk preferences for all decision makers. The exponential function is more flexible because the risk-tolerance parameter *(R)* allows different risk preferences to be modeled. Defining your own utility curve provides the greatest flexibility, but also requires some programming.

- Read the example on Pages 546 – 550.