CHAPTER

11a

Duxbury Thomson Learning

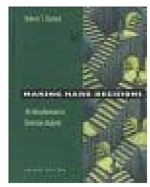
Making Hard Decision

Third Edition

MONTE CARLO SIMULATION

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

FALL 2003



By

Dr . Ibrahim. Assakkaf

ENCE 627 – Decision Analysis for Engineering

Department of Civil and Environmental Engineering

University of Maryland, College Park

Simulation

Methodology of Modeling Uncertainty

The Methodology of Modeling Uncertainty is described in five chapters that mainly concentrating on how to model uncertainty using probabilities and information as follows:

Detailed Steps

- Probability Basics: reviews fundamental probability concepts. — ***Chapter 7***
- Subjective probability: translates beliefs & feelings about uncertainty in probability for use in decision modeling. — ***Chapter 8***
- Theoretical Probability Models: helps with representing uncertainty in decision modeling — ***Chapter 9***
- Using Data: uses historical data for developing probability distributions — ***Chapter 10***
- **Monte Carlo Simulation: to give the decision-maker a fair idea about the probabilities associated with various outcomes.** — ***Chapter 11***
- Value of Information: explores the value of information within the decision-analysis framework. — ***Chapter 12***

Simulation

- **Simulation is a very powerful and widely used management science technique for the analysis and study of different types of systems.**
- **Because of complexity, stochastic relations, and so on, not all real-world problems can be represented adequately in a model form.**
- **Attempts to use analytical models for such systems usually require so many simplifying assumptions that the solutions are likely to be inferior of inadequate for implementation.**
- **Often, in such instances, the only alternative form of modeling and analysis available to the decision maker is simulation.**

Simulation

- ***Simulation*** may be defined as a technique that imitates the operation of a real-world system as it evolves over time. This is normally done by developing a simulation model.
- A ***simulation model*** take the form of a set of assumptions about the operation of the system, expressed as mathematical or logical relations between the objects of interest in the system.
- In contrast to the exact mathematical solutions available with most analytical models, the simulation process involves executing or running the model through time, usually on a computer, to generate representative samples of the measures of performance. In this respect, simulation may be seen as a sampling experiment on the real system, with the results being sample points.
- To obtain the best estimate of the mean of the measure of performance, we average the sample results. Clearly, the sample points we generate, the better our estimate will be. However, other factors, such as the starting conditions of the simulation, the length of the period being simulated, and the accuracy of the model itself, all have a bearing on how good our final estimate will be.

Advantages and Disadvantages

- The major advantage of simulation is that simulation theory is relatively straightforward.
- In general, simulation methods are easier to apply than analytical methods. Whereas analytical models may require us to make many simplifying assumptions, simulation models have few such restrictions, thereby allowing much greater flexibility in representing the real system.
- Once a model is built, it can be used repeatedly to analyze different policies, parameters, or designs.

Advantages and Disadvantages

- **Example:**

  If a business firm has a simulation model of its inventory system, various inventory policies can be tried on the model rather than taking the chance of experimenting on the real-world system. However, it must be emphasized that simulation is not an optimizing technique. It is most often used to analyze "what if" types of questions. Optimization with simulation is possible, but it is usually a slow process. Simulation can also be costly. However, with the development of special-purpose simulation languages, decreasing computational cost, and advances in simulation methodologies, the problem of cost is becoming less important.

Basic Terminology

In order to model a system, we must understand the concept of a system. Among the many different ways of defining a system, the most appropriate definition for simulation problems is the one proposed by Schmidt and Taylor (1970).

■Definition:

- **A *system* is a collection of entities that act and interact toward the accomplishment of some logical end.**
- **In practice, however, this definition generally tends to be more flexible. The exact description of the system usually depends on the objective of the simulation study. For example, what may be a system for a particular study may be only a subset of the overall system for another.**
- **Systems generally tend to be dynamic–their status changes over time. To describe this status, we use the concept of the state of a system.**

Basic Terminology (cont'd)

Definition:

- The ***state*** of a system is the collection of variables necessary to describe the status of the system at any given time.
- As an example of a system, let us consider a bank. Here, the system consists of the servers and the customers waiting in line or being served. As customers arrive or depart, the status of the system changes. To describe these changes in status, we require a set of variables called the *state variables*.

Example:

- The bank's customers may be described as the entities, and the characteristics of the customers (such as the occupation of a customer) may be defined as the attributes.
- Systems may be classified as discrete or continuous.

Basic Terminology (cont'd)

Definition:

- A ***discrete system*** is one in which the state variables change only at discrete or countable points in time.
- A bank is an example of a discrete system, since the state variables change only when a customer arrives or when a customer finishes being served and departs. These changes take place at discrete points in time.

Definition:

- A ***continuous system*** is one in which the state variables change continuously over time.
- A chemical process is an example of a continuous system. Here, the status of the system is changing continuously over time. Such systems are usually modeled using differential equations. We do not discuss any continuous systems in this chapter.
- There are two types of simulation models, static and dynamic.

Basic Terminology (cont'd)

Definition:

- A ***static simulation model*** is a representation of a system at a particular point in time. We usually refer to a static simulation as a *Monte Carlo Simulation*.

Definition:

- A ***dynamic simulation*** is a representation of a system as it evolves over time.
- Within these two classifications, a simulation may be deterministic or stochastic.

Definition:

- A ***deterministic simulation model*** is one that contains no random variables; e.g. profit = 45 – 20 = $25. A *stochastic simulation model* contains one or more random variables; e.g. profit = 45 – 20, 21, 22, 23 = either $25 or $24 or $23 or $ 22.
- Discrete and continuous simulation models are similar to discrete stochastic models. Such models are called discrete–event simulation models. *Discrete-event simulation* concerns the modeling of a stochastic system as it evolves over time by a representation in which state variables change only at discrete points in time.

An Example of a Discrete-Event Simulation

- A single-server queuing system. Customers arrive into this system from some population and either go into service immediately if the server is idle or join a waiting line (queue) if the server is busy. Examples of this kind of a system are:
- a one-person barber shop,

- a small grocery store with only one checkout counter, and

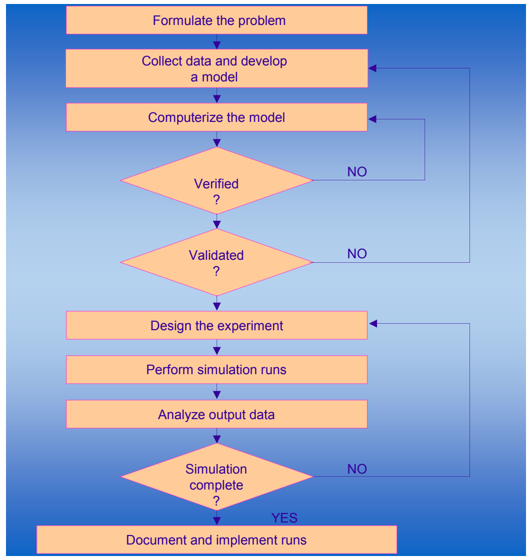
- a single ticket counter at an airline terminal.

The Simulation Process

Simulation Process and Steps

- ***Statement of objectives***
- ***Model Development***
- ***Data Collection and Preparation***
- ***Solving the Model using Computer Program***
- ***Verification***
- ***Validation***
- ***Documentation***

Steps of Simulation



Statement of the Objectives

- The initial stage of any scientific study, including a simulation project, requires an explicit ***statement of the objectives*** of the study. This should include the questions to be answered, the hypothesis to be tested, and the alternatives to be considered.
- Without a clear understanding and description of the problem, the chances of successful completion and implementation are greatly diminished. Also in this step, we address issues such as the performance criteria, the model parameters, and the identification and definition of the state variables.
- It is, of course, very likely that the initial formulation of the problem will undergo many modifications as the study proceeds and as we learn more about the situation being studied. Nevertheless, a clear initial statement of the objectives is essential.

Model Development and Data Collection

- The next stage is the *development* of the model and the *collection of data*.
- The development of the model is probably the most difficult and critical part of a simulation study.
- Here, we try to represent the essential features of the systems under study by mathematical or logical relations.
- There are a few firm rules to guide an analyst on how to go about this process.
- In many ways, this is as much an art as a science. However, most experts agree that the best approach is to start with a simple model and make it more detailed and complex as one learns more about the system.

Developing a Computer Program

- Having developed the model, we next put it into a form in which it can be analyzed on the computer.
- This usually involves *developing a computer program* for the model. One of the key decisions here is the choice of the language. As noted earlier, the special-purpose languages require less programming than the general-purpose languages but are less flexible and tend to require longer computer running times. In either case, the programming part of the study is likely to be a time-consuming process, since simulation programs tend to be long and complex.

Verification

- Once the program has been developed and debugged, we determine whether the program is working properly.

- In other words, is the program doing what it is suppose to do?

- This process is called the ***verification*** step and is usually difficult, since for most simulations, we will not have any results with which to compare the computer output.

Validation

- If we are satisfied with the program, we now move to the validation stage.
- This is another critical part of a simulation study. In this step, we validate the model to determine whether it realistically represents the system being analyzed and whether the results from the model will be reliable.
- As with the verification stage, this is generally a difficult process. Each model presents a different challenge. However, there are some general guidelines that one can follow.

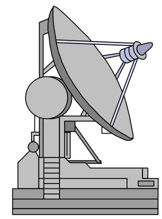
Validation (cont'd)

- If we are satisfied at this stage with the performance of the model, we can use the model to conduct the experiments to answer the questions at hand. The data generated by the simulation experiments must be collected, processed, and analyzed.
- The results are analyzed not only as the solutions to the model but also in terms of statistical reliability and validity. Finally, after the results are processed and analyzed, a decision must be made whether to perform any additional experiments.

Monte Carlo Simulation

Monte Carlo Simulation

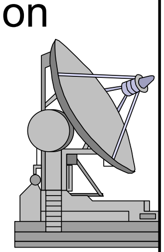
- Started in the early 1940's for the purpose of developing inexpensive techniques for testing engineering systems by imitating their real behavior.



- These methods are commonly called Monte Carlo simulation techniques.

Monte Carlo Simulation

- The principle behind the methods is to develop an analytical model, which is computer based, that predicts the behavior of a system. Then, the model is evaluated, and therefore the behavior is predicted, several times. Each evaluation (or called simulation cycle) is based on some randomly selected conditions for the input parameters of the system.



Monte Carlo Simulation

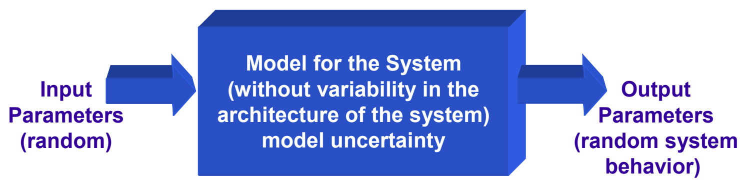
- Certain analytical tools are used to assure the random selection of the input parameters according to their respective probability distributions for each evaluation. As a result, several predictions of the behavior are obtained. Then, statistical methods are used to evaluate the moments and distribution type for the system's behavior.

General Procedure

- The analytical and computational steps that are needed for performing Monte Carlo simulation are:
  - Definition of the system
  - Generation of random numbers
  - Generation of random variables
  - Evaluation of the model N times
  - Statistical analysis of the resulting behavior
  - Study of efficiency and convergence

System Definition

- The definition of the system should includes its boundaries, input parameters, output (or behavior) measures, architecture, and models that relate the input parameters and architecture to the output parameters.

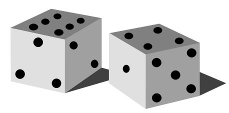


Generation of Random Numbers

- Random numbers are real values, if normalized using the largest possible value, result in real values in the range [0,1].
- Random numbers have a uniform distribution on the range [0,1].
- A set of random numbers should also satisfy the condition of non-correlation for the purpose of simulation use.

Generation of Random Numbers

- Significance
  - Their transformation into real values that follow any distribution of interest. They constitute the basis for random variable generation



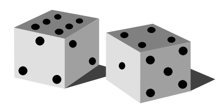
- Types
  - Mechanical
  - Tabulated
  - Computer based (recursive functions) using a seed

Table of Random Numbers in the Range [0,1]

0.538246	0.181648	0.172614	0.450166	0.293027	0.030195	0.757836	0.915061
0.663357	0.368934	0.516388	0.656254	0.284258	0.906335	0.329788	0.054487
0.035771	0.053784	0.424573	0.942479	0.293872	0.326815	0.862351	0.358055
0.51356	0.165508	0.667312	0.878444	0.414203	0.100839	0.555287	0.685601
0.880006	0.069305	0.85441	0.371911	0.751341	0.128446	0.678679	0.514995
0.880006	0.069305	0.85441	0.371911	0.751341	0.128446	0.678679	0.514995
0.748794	0.902497	0.629615	0.662531	0.932879	0.018376	0.683876	0.55481
0.115441	0.207278	0.887853	0.812124	0.082143	0.939258	0.666874	0.582525
0.953369	0.543997	0.806486	0.707493	0.503949	0.489926	0.774467	0.248617
0.2436	0.537111	0.181388	0.619277	0.131852	0.131876	0.361814	0.582682
0.610186	0.41158	0.339972	0.080869	0.429448	0.82277	0.63269	0.863227
0.848375	0.043973	0.071429	0.713405	0.56201	0.71605	0.53662	0.357681
0.102922	0.201752	0.61727	0.416471	0.371492	0.633301	0.857578	0.483474
0.009326	0.912932	0.11385	0.3316	0.852807	0.626191	0.035676	0.581386
0.801494	0.365068	0.54875	0.480788	0.032959	0.906331	0.291263	0.706212
0.682049	0.946008	0.960047	0.830463	0.186225	0.123762	0.674147	0.012839

Generation of Random Variables

- Inverse Transformation Method
  - A random number u is first generated in the range [0,1]
  - Then the value (x) of a generated continuous random variable, X, is determined as follows:

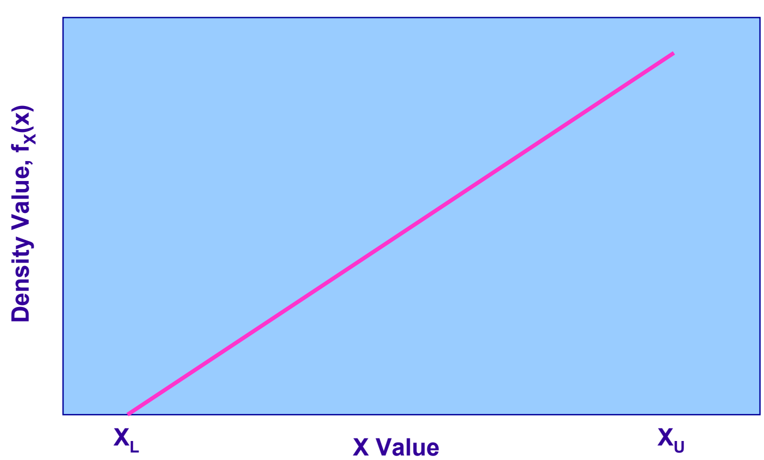


$x = F_X^{-1}(u)$ = **the inverse of the cumulative distribution function of the random variable X evaluated at u.**

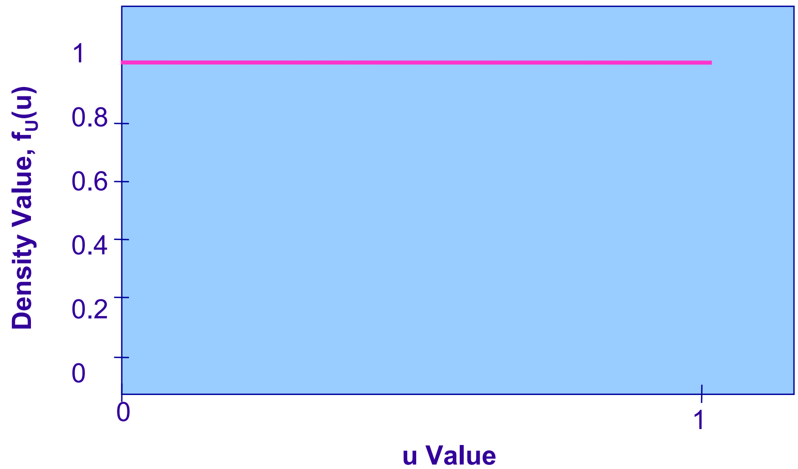
Since the range of $F_X(x)$ is in the range [0,1], a unique value for x is obtained all the time in each simulation cycle.

Triangular Distribution

- Triangular Density Function



Uniform Density Function



Cumulative Function

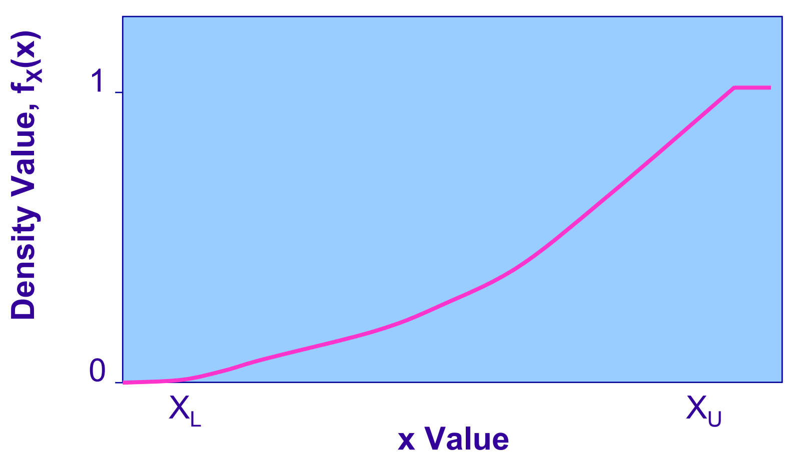


Illustration of inverse transformation

Normal Distribution

- For mean μ and standard deviation s, the uniform variate is first transformed to the standard normal deviate z, which is then transformed to normal deviate x by:

$$x = \mu + z\sigma$$

Normal Distribution (cont'd)

- Consider the case where the sample consists of the following:

U = {0.82, 0.43, 0.71, 0.18, 0.66}

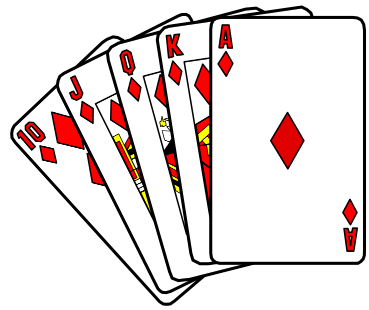
- To find the corresponding z values, enter the standard normal table for a probability of 0.82, for example, and read the z value (z = 0.915). Continuing for each ui value yields:

Z = {0.915, -0.176, 0.556, -0.915, 0.413}

- Then use x = m + z s to obtain the values of X.

Other Methods for Generating Random Variables

- Specialized
- Efficient
- Automated



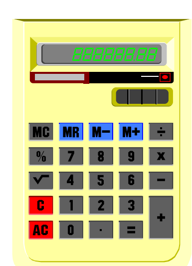
Evaluation of Model

Substitute the generated input random variables (X_i) into an analytical model, g($\underline{X}$) to predict a response Y. The model can be a simple education or a complex computer code.

$$Y = g(X_1, X_2, X_3, \ldots, X_n)$$

This process is repeated N times.

- Noncorrelated random variables
- Correlated random variables



Statistical Analysis of Results

- Mean response:

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{n} Y_i$$

- Variance:

$$\mathrm{Var}(Y) = \frac{1}{N-1}\left[\sum_{i=1}^{N} Y_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} Y_i\right)^2\right]$$

- Variance of estimated mean:

$$\mathrm{Var}(\bar{Y}) = \frac{1}{N}\frac{1}{N-1}\left[\sum_{i=1}^{N} Y_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} Y_i\right)^2\right]$$