Solution Homework Set #2

ENCE 627 – Decision Analysis for Engineering – Fall 2003

Assigned T, 9/16 Due T, 6/23

Problem 1

Textbook (CR): 8.1

***** SOLUTION *****

Subjective probability can be defined as the degree of belief, uncertainty in one's mind, or a willingness to bet or accept lotteries.

Problem 2

Textbook (CR): 8.3

***** SOLUTION *****

Assessing a discrete probability requires only one judgment. Assessing a continuous probability distribution can require many subjective judgments of interval probabilities, cumulative probabilities, or fractiles in order to sketch out the CDF. Even so, the fundamental probability assessments required in the continuous case are essentially the same as in the discrete case.

Problem 3

Textbook (CR): 8.10

***** SOLUTION *****

Let

$LW =$ Napoleon wins

$NL =$ Napoleon loses

$P\,\&\,E =$ Prussians and English have joined forces

Using Baye's theorem:

$$\mathrm{P}(NW \mid P\,\&\,E) = \frac{\mathrm{P}(P\,\&\,E \mid NW)\mathrm{P}(NW)}{\mathrm{P}(P\,\&\,E)}$$

$$\mathrm{P}\left(NL \mid P\,\&\,E)\right) = \frac{\mathrm{P}(P\,\&\,E \mid NL)\mathrm{P}(NL)}{\mathrm{P}(P\,\&\,E)}$$

Therefore,

$$\frac{\mathrm{P}(NW \mid P\,\&\,E)}{\mathrm{P}(NL \mid P\,\&\,E)} = \frac{\mathrm{P}(P\,\&\,E \mid NW)}{\mathrm{P}(P\,\&\,E \mid NL)}\frac{\mathrm{P}(NW)}{\mathrm{P}(NL)}$$

We have that P(NW) = 0.90 and P(NW | $P\&E$) = 0.60, and so P(NL) = 0.10 and P(NL | $P\&E$) = 0.40. Thus, we can substitute:

$$\frac{0.60}{0.40} = \frac{P(P\,\&\,E \mid NW)}{P(P\,\&\,E \mid NL)}\frac{0.90}{0.10}$$

Or

$$\frac{\mathrm{P}(P\,\&\,E \mid NW)}{\mathrm{P}(P\,\&\,E) \mid NL)} = \frac{0.60(0.1)}{0.4(0.9)} = 0.1667 = \frac{1}{6}$$

Problem 4

Textbook (CR): 8.27

***** SOLUTION *****

(a) Prefers A because the options for which the probability of winning is known.
(b) Prefers D because the options for which the probability of winning is known.
(c) Choosing A and D may appear to be consistent because both of these involve known probabilities. However, consider the EMVs for the lotteries and the implied values for P(Blue). If A is preferred to B, then

$$\text{EMV(A)} > \text{EMV(B)}$$
$$\frac{1}{3}(1000) > \text{P(Blue)}\,(1000)$$
$$\text{P(Blue)} < \frac{1}{3}\ .$$

However, if D is preferred to C, then

$$\text{EMV(D)} > \text{EMV(C)}$$
$$\text{P(Blue)}\,(1000) + \text{P(Yellow)}\,(1000) > \frac{1}{3}(1000) + \text{P(Yellow)}\,(1000)$$
$$\text{P(Blue)} > \frac{1}{3}\ .$$

The inconsistency arises because it clearly is not possible to have both P(Blue) < $\frac{1}{3}$ and P(Blue) > $\frac{1}{3}$. (Exactly the same result obtains if we use the utility of \$1000, instead of the dollar value.)

Problem 5

The following cumulative probabilities for a movie star's age were evaluated based on a personal and subjective assessment:

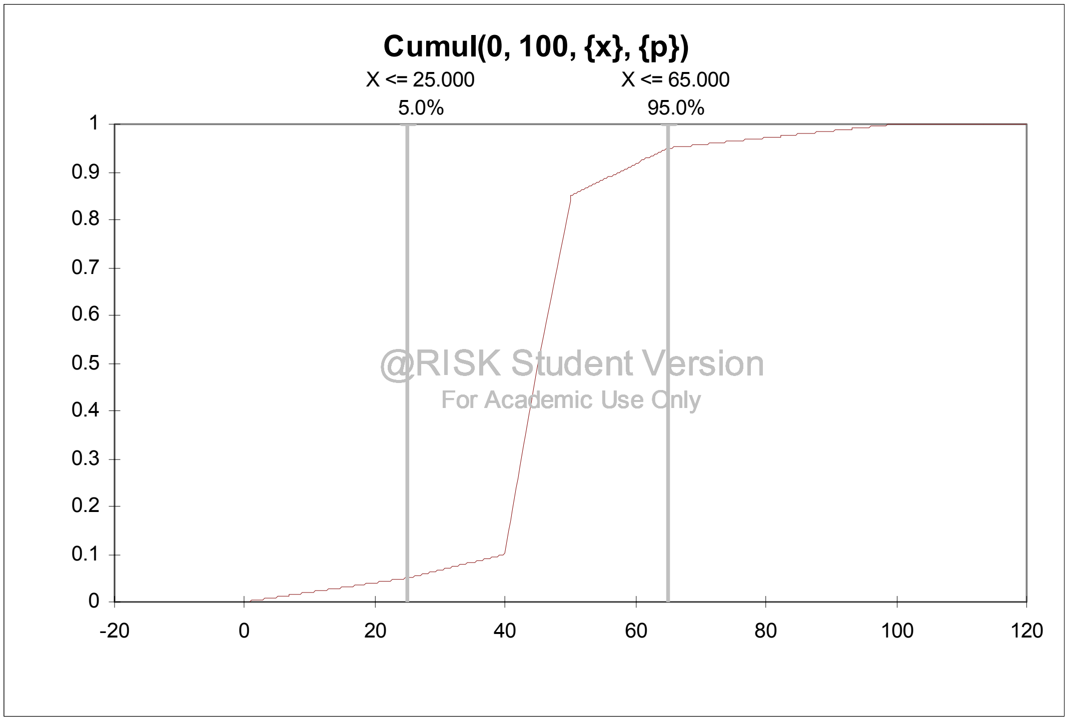
$$P\,(\text{Age} \le 25) = 0.05$$
$$P\,(\text{Age} \le 40) = 0.10$$
$$P\,(\text{Age} \le 45) = 0.50$$
$$P\,(\text{Age} \le 50) = 0.85$$
$$P\,(\text{Age} \le 65) = 0.95$$

Using @RISK and Excel, and assuming that a movie star can live up to 100 years:

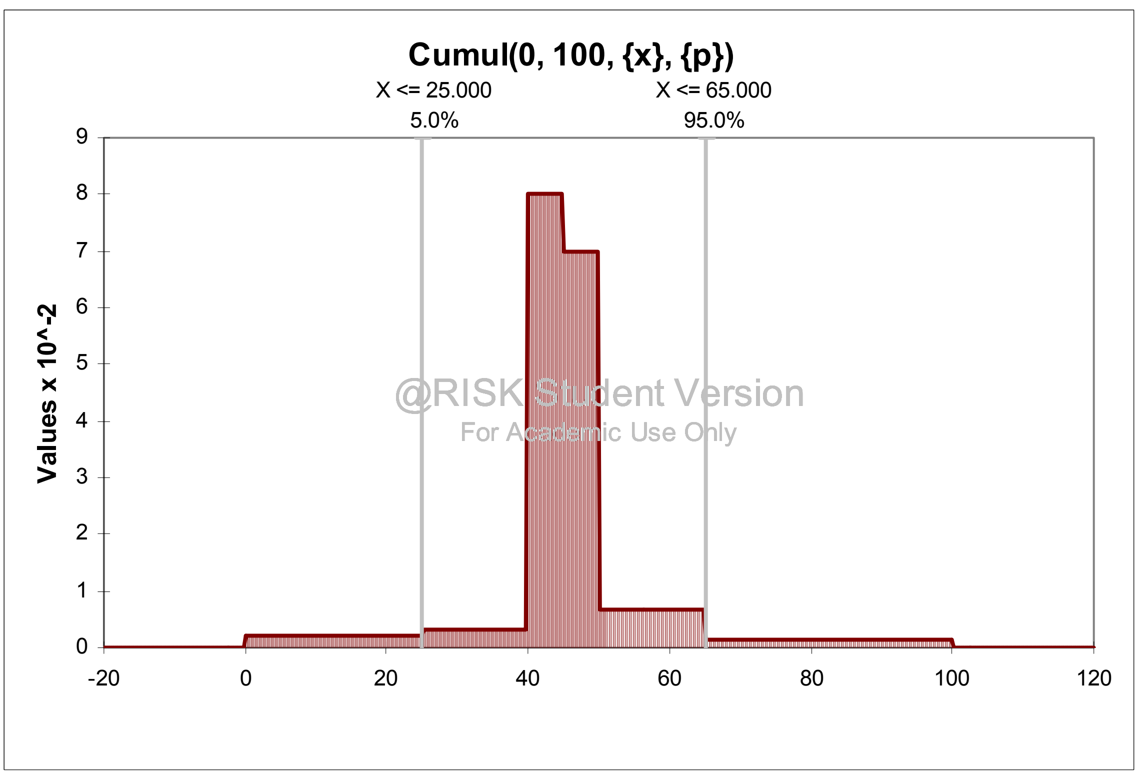
(a) Draw the cumulative distribution function (CDF),
(b) Draw the density function,
(c) Find the probability that the age of a movie star will be between 35 and 55 years,
(d) What is the probability that a movie star's age is greater than 60 years,
(e) What is the probability that a movie star's age is 48 years,
(f) Determine the age values so that there is an 18% chance that a movie star's age will be below the first age value and an 18% chance that the age value will be above the second age value.

***** SOLUTION *****

(a) Cumulative distribution function (CDF):



(b) <u>Mass Density Function:</u>



(c) P (35 ≤ Age ≤ 55) = 0.8833– 0.833 – 0.80

(d) P (Age > 60) = 0.083

(e) By definition for continuous probability, P (Age = 48) = 0

(f) 41 and 50 (49.5714)