Solution to Homework Set #1
ENCE 627 – Decision Analysis for Engineering – Fall 2003

Assigned T, 9/9   Due T, 6/16

<u>Problem 1</u>

The following concrete strength data (in ksi) were collected using an ultrasonic non-destructive testing method at different locations of an existing structure:
3.5, 3.2, 3.1, 3.5, 3.6, 3.2, 3.4, 2.9, 4.1, 2.6, 3.3, 3.5, 3.9, 3.8, 3.7, 3.4, 3.6, 3.5, 3.5, 3.7, 3.6, 3.8, 3.2, 3.4, 4.2, 3.6, 3.1, 2.9, 2.5, 3.5, 3.4, 3.2, 3.7, 3.8, 3.4, 3.6, 3.5, 3.2, 3.6, and 3.8

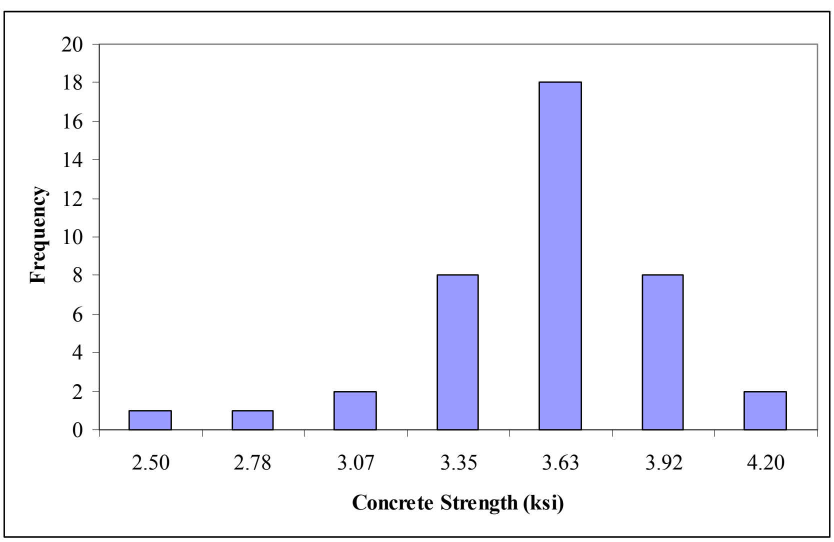
(a) Plot a histogram and a frequency diagram for concrete strength.
(b) Determine the central tendency measures, i.e., the average value, median and mode.
(c) Determine the dispersion measures, i.e., the variance, standard deviation and coefficient of variation.

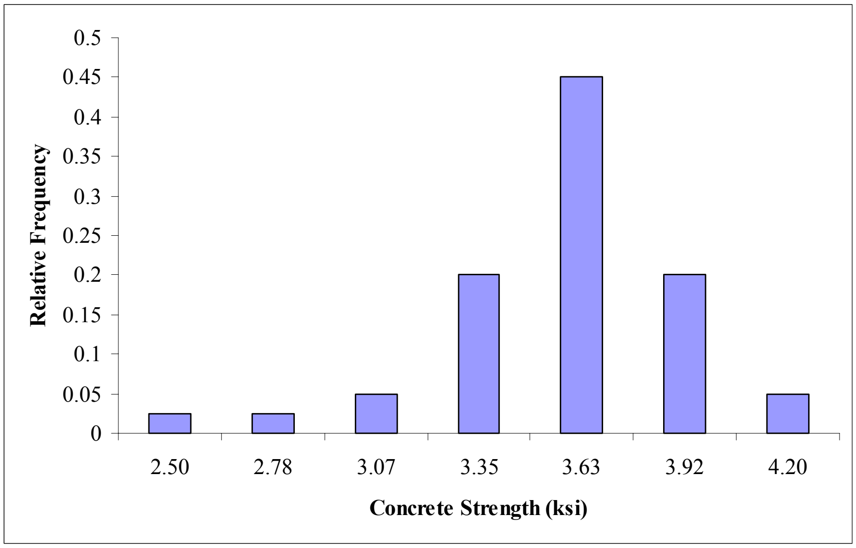
***** SOLUTION *****

Sorted Data:

2.5	3.2	3.5	3.7
2.6	3.3	3.5	3.7
2.9	3.4	3.5	3.7
2.9	3.4	3.5	3.8
3.1	3.4	3.6	3.8
3.1	3.4	3.6	3.8
3.2	3.4	3.6	3.8
3.2	3.5	3.6	3.9
3.2	3.5	3.6	4.1
3.2	3.5	3.6	4.2

(a) <u>Histogram and Frequency Diagrams:</u> $k = 1 + 3.3\log_{10}(40) = 6.28 = 6$





(b) Central Tendency Measures:

Average (Mean)	3.45 ksi
Median	3.5 ksi
Mode	3.5 ksi

(c) Dispersion Measures:

Var	0.1241 ksi
St. Dev	0.3523 ksi
COV	0.1021 ksi

Problem 2

If the sample space $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{0, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 9\}$, $C = \{2, 3, 4, 5\}$, and $D = \{1, 6, 7\}$, list the elements of the sets corresponding to the following events:

(a) $A \cup C$

(b) $A \cap B$

(c) $\overline{C}$

(d) $\left(\overline{C} \cap D\right) \cup B$

(e) $\overline{S \cap C}$

***** SOLUTION *****

a) $A \cup C$

$$A \cup C = \{0,2,3,4,5,6,8\}$$

b) $A \cap B$

$$A \cap B = \varnothing = \text{empty set or impossible event}$$

c) $\overline{C}$

$$\overline{C} = \{0,1,6,7,8,9\}$$

d) $\left(\overline{C} \cap D\right) \cup B$

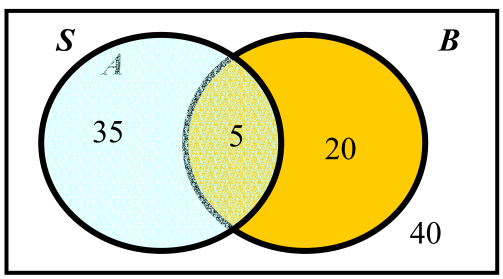
$$\left(\overline{C} \cap D\right) \cup B = \{1,6,7\} \cup B = \{1,3,5,6,7,9\}$$

e) $\overline{S \cap C}$

$$\overline{S \cap C} = \overline{S} \cup \overline{C} = \varnothing \cup \overline{C} = \{0,1,6,7,8,9\}$$

Problem 3

Refer to the Venn diagram shown below for event *A* and *B* in the sample space *S*. Find each of the indicated probabilities.



(a) $\mathrm{P}(A)$ and $\mathrm{P}\left(\overline{A}\right)$

(b) $\mathrm{P}(B)$ and $\mathrm{P}\left(\overline{B}\right)$

(c) $\mathrm{P}(A \cap B)$ and $\mathrm{P}\left(\overline{A} \cap \overline{B}\right)$

(d) $\mathrm{P}\left(\overline{A} \cap B\right)$ and $\mathrm{P}\left(A \cap \overline{B}\right)$

(e) $\mathrm{P}(A \cup B)$ and $\mathrm{P}\left(\overline{A \cup B}\right)$

***** SOLUTION *****

(f) $\mathrm{P}(A)$ and $\mathrm{P}\left(\overline{A}\right)$

$$\mathrm{P}(A) = \frac{35+5}{35+5+20+40} = \frac{40}{100} = 0.4$$

$$\mathrm{P}\left(\overline{A}\right) = 1 - 0.4 = 0.6$$

(g) $\mathrm{P}(B)$ and $\mathrm{P}\left(\overline{B}\right)$

$$\mathrm{P}(B) = \frac{5+20}{35+5+20+40} = \frac{25}{100} = 0.25$$

$$\mathrm{P}\left(\overline{B}\right) = 1 - 0.25 = 0.75$$

(h) $P(A \cap B)$ and $P(\overline{A} \cap \overline{B})$

$$P(A \cap B) = \frac{5}{35+5+20+40} = \frac{5}{100} = 0.05$$

$$P(\overline{A} \cap \overline{B}) = \frac{40}{35+5+20+40} = \frac{40}{100} = 0.4$$

(i) $P(\overline{A} \cap B)$ and $P(A \cap \overline{B})$

$$P(\overline{A} \cap B) = \frac{20}{100} = 0.20 \quad \text{and } P(A \cap \overline{B}) = \frac{35}{100} = 0.35$$

(j) $P(A \cup B)$ and $P(\overline{A \cup B})$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.40 + 0.25 - 0.05 = 0.6$$

$$P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.60 = 0.40, \text{ Also, } P(\overline{A \cup B}) = P(\overline{A} \cap \overline{B}) = 0.4$$

Problem 4

Textbook (CR): 7.4 **Solution:**

P(A or B) = P(A and B) + P(A and $\overline{B}$) + P($\overline{A}$ and B)
= 0.12 + 0.53 + 0.29 = 0.94

or P(A or B) = P(A) + P(B) - P(A and B)
= 0.41 + 0.65 - 0.12 = 0.94

or P(A or B) = 1 - P($\overline{A}$ and $\overline{B}$) = 1 - 0.06 = 0.94

Problem 5

Textbook (CR): 7.15 **Solution:**

P(offer) = 0.50

P(good interview | offer) = 0.95

P(good interview | no offer) = 0.75

P(offer | good interview) = P(offer | good)

$$= \frac{\text{P(good | offer) P(offer)}}{\text{P(good | offer) P(offer) + P(good | no offer) P(no offer)}}$$

$$= \frac{0.95\ (0.50)}{0.95\ (0.50) + 0.75\ (0.50)}$$

= 0.5588

Problem 6

Textbook (CR): 7.16 (only part a) **Solution:**

a. E(*X*) = 0.05 (1) + 0.45 (2) + 0.30(3) + 0.20(4)
= 0.05 + 0.90 + 0.90 + 0.80
= 2.65

$$\mathrm{Var}(X) = 0.05\,(1-2.65)^2 + 0.45\,(2-2.65)^2 + 0.30(3-2.65)^2 + 0.20(4-2.65)^2$$
$$= 0.05\,(2.72) + 0.45\,(0.42) + 0.30(0.12) + 0.20(1.82)$$
$$= 0.728$$

$$\sigma_X = \sqrt{0.728} = 0.853$$

Problem 7

Textbook (CR): 7.19 **Solution:**

a. E(Revenue from A)

= \$3.50 E(Unit sales)
= \$3.50 (2000)
= \$7000

Var(Revenue from A)

= 3.50^2 Var(Unit sales)
= 3.50^2 (1000)
= 12,250 "dollars squared"

b. E(Total revenue)

= \$3.50 (2000) + \$2.00 (10,000) + \$1.87 (8500)
= \$42,895

Var(Total revenue)

= 3.50^2 (1000) + 2.00^2 (6400) + 1.87^2 (1150)
= 41,871 "dollars squared"

Problem 8

For the following probability density function:

$$f_X(x) = \begin{cases} kx & \text{for } 0 \le x \le 1 \\ k & \text{for } 1 < x \le 2 \end{cases}$$

(a) Find the value k that is necessary to make the probability density function legitimate,
(b) Graph both the density and the cumulative functions,
(c) Determine the mean, variance, standard deviation, and coefficient of variation.
(d) Find the following probabilities:
$P(X = 1.5)$, $P(X > 0.5)$, and $P(1 \le X \le 2)$

***** SOLUTION *****

(a) <u>*k* value: see next page.</u>

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 kx\,dx + \int_1^2 k\,dx$$

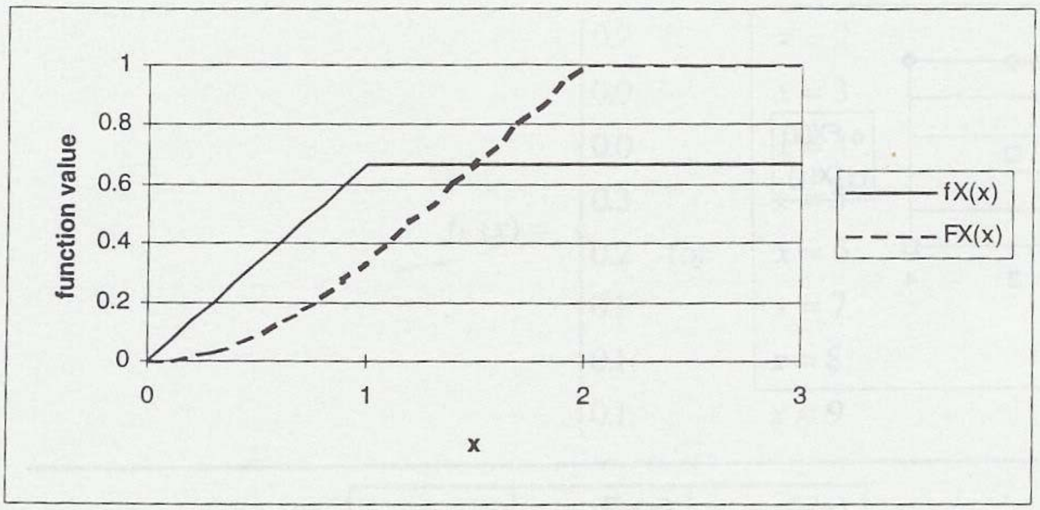
$$= k\,\frac{x^2}{2}\Big|_0^1 + k\,x\Big|_1^2 = \frac{k}{2} + k = \frac{3}{2}\,k$$

$$k = \frac{2}{3}$$

$$F(x) = \int_0^x k\,t\,dt = \int_0^x \frac{2}{3}\,t\,td = \frac{2}{3}\frac{t^2}{2}\Big|_0^x = \frac{1}{3}\,x^2 \quad \text{fo r } (0 \le x \le 1)$$

$$F(x) = \frac{1}{3} + \int_1^x k\,dt = \frac{1}{3} + \int_1^x \frac{2}{3}\,dt = \frac{1}{3} + \frac{2}{3}\,t\Big|_1^x = \frac{1}{3} + \frac{2}{3}\,(x-1) \quad \text{for } (1 < x \le 2)$$

(b) Graphs of the density and the cumulative functions:



(c) Mean, variance, standard deviation, and coefficient of variation:

$$f_X(x) = \begin{cases} \frac{2}{3}x & \text{for } 0 \le x \le 1 \\ \frac{2}{3} & \text{for } 1 < x \le 2 \end{cases}$$

$$\mu = \int_0^1 x\left(\frac{2}{3}x\right)dx + \int_1^2 x\left(\frac{2}{3}\right)dx = \frac{11}{9}$$

or from geometry in Problem 3–19

$$\mu = (\frac{1}{2})(\frac{2}{3}) + (\frac{2}{3})(\frac{3}{2}) = \frac{2}{9} + 1 = \frac{11}{9}$$

$$\sigma^2 = \int_0^1 \left(x - \frac{11}{9}\right)^2 \left(\frac{2}{3}x\right)dx + \int_1^2 \left(x - \frac{11}{9}\right)^2 \left(\frac{2}{3}\right)dx = 5.1173$$

$$\sigma = 2.2621$$

(d) <u>Finding Probabilities:</u>

$P(1.5) = 0$, by definition of continuous probability distribution.

$$P\,(X > 0.5) = 1 - P(X \le 0.5) = 1 - F_X(0.5) = 1 - \frac{1}{3}(0.5)^2 = 0.92$$

$$P\,(1 \le X \le 2) = F_X(2) - F_X(1) = \left[\frac{1}{3} + \frac{2}{3}(2-1)\right] - \frac{1}{3}(1)^2 = \frac{2}{3} = 0.67$$