University of Maryland, College Park
Department of Civil and Environmental Engineering

Quiz 3 Solution, **closed book & notes, for 15 minutes**
October 12, 2001

ENCE 302
Probability and Statistics for Civil Engineers **Name:___________________**

Problem 1

A high-rise building can be structurally damaged by failure in the foundation or in the main structure. The corresponding failure probabilities of the foundation and the main structure are estimated to be 0.07 and 0.02, respectively. Also, if there is foundation failure, then the probability that the main structure will suffer some structural damage is 0.6.

a. What is the probability of damage to the building?
b. If damage in the foundation and in main structure is independent events, what is the probability of damage to the building?

***** SOLUTION *****

The given information can be summarized as follows:

$\mathrm{P}(F) = \text{Probability failure in foundation} = 0.07$

$\mathrm{P}(S) = \text{Probability of failure in main structure} = 0.02$

$\mathrm{P}(SF) = \text{Probability of failure of main structure given that failure has occured in foundation} = 0.6$

a)

The probability of damage to the building =

$$= \mathrm{P}(F \cup S) = \mathrm{P}(F) + \mathrm{P}(S) - \mathrm{P}(F \cap S)$$
$$= \mathrm{P}(F) + \mathrm{P}(S) - \mathrm{P}(S \mid F)\mathrm{P}(F)$$
$$= 0.07 + 0.02 - 0.6 \times 0.07 = 0.048$$

b)

If F and S are statistically independent, then,
the probability of damage to the building =

$$= \mathrm{P}(F \cup S) = \mathrm{P}(F) + \mathrm{P}(S) - \mathrm{P}(F \cap S)$$
$$= \mathrm{P}(F) + \mathrm{P}(S) - \mathrm{P}(F)\mathrm{P}(S)$$
$$= 0.07 + 0.02 - 0.07 \times 0.02 = 0.0886$$

Formulas

Rule Type	Operations
Identity Laws	$A \cup \varnothing = A,\ A \cap \varnothing = \varnothing,\ A \cup S = S,\ A \cap S = A$
Idem potent Laws	$A \cup A = A,\ A \cap A = A$
Complement Laws	$A \cup \overline{A} = S,\ A \cap \overline{A} = \varnothing,\ \overline{\overline{A}} = A,\ \overline{S} = \varnothing,\ \overline{\varnothing} = S$
Commutative Laws	$A \cup B = B \cup A,\ A \cap B = B \cap A$
Associative Laws	$(A \cup B) \cup C = A \cup (B \cup C), (A \cap B) \cap C = A \cap (B \cap C)$
Distributive Laws	$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
De Morgan's Law	$\overline{(A \cup B)} = \overline{A} \cap \overline{B}, \overline{(E_1 \cup E_2 ... \cup E_n)} = \overline{E_1} \cap \overline{E_2} ... \cap \overline{E_n}$ $\overline{(A \cap B)} = \overline{A} \cup \overline{B}, \overline{(E_1 \cap E_2 \cap ... \cap E_n)} = \overline{E_1} \cup \overline{E_2} \cup ... \cup \overline{E_n}$
Combinations of Laws	$\overline{(A \cup (B \cap C))} = \overline{A} \cap \overline{(B \cap C)} = (\overline{A} \cap \overline{B}) \cup (\overline{A} \cap \overline{C})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad P(B) \neq 0$$

Properties of Conditional Probability

1. The complement of an event:

$$P(\overline{A} \mid B) = 1 - P(A \mid B)$$

2. The multiplication rule for two events *A* and *B*:

$$P(A \cap B) = P(A \mid B)P(B) \qquad \text{if } P(B) \neq 0$$
$$P(A \cap B) = P(B \mid A)P(A) \qquad \text{if } P(A) \neq 0$$

Properties of Conditional Probability

3. The multiplications rule for three events *A*, *B*, and *C*:

$$P(A \cap B \cap C) = P(A \mid (B \cap C))P(B \mid C)P(C) = P((A \cap B) \mid C)P(C)$$
$$\text{if } P(C) \neq 0 \text{ and } P(B \cap C) \neq 0$$

Properties of Conditional Probability

4. For mutually independent events *A* and *B*:

$$P(A \mid B) = 0$$
$$P(B \mid A) = 0$$

5. For statistically independent events *A* and *B*:

$$P(A \mid B) = P(A)$$
$$P(B \mid A) = P(B)$$
$$P(A \cap B) = P(A)P(B)$$