CHAPTER

Probability, Statistics, and Reliability for Engineers and Scientists

Second Edition

SIMULATION

• A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

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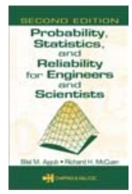

Probability and Statistics for Civil Engineers

Department of Civil and Environmental Engineering

University of Maryland, College Park

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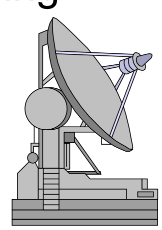
Introduction

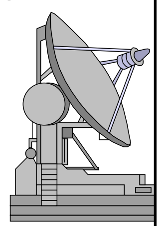
- Started in the early 1940's for the purpose of developing inexpensive techniques for testing engineering systems by imitating their real behavior.



- These methods are commonly called Monte Carlo simulation techniques.

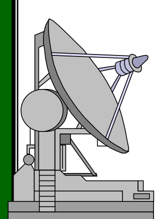
Introduction (cont'd)

- The principle behind the methods is to develop an analytical model, which is computer based, that predicts the behavior of a system. Then, the model is evaluated, and therefore the behavior is predicted, several times. Each evaluation (or called simulation cycle) is based on some randomly selected conditions for the input parameters of the system.



Introduction (cont'd))

- Certain analytical tools are used to assure the random selection of the input parameters according to their respective probability distributions for each evaluation. As a result, several predictions of the behavior are obtained. Then, statistical methods are used to evaluate the moments and distribution type for the system's behavior.

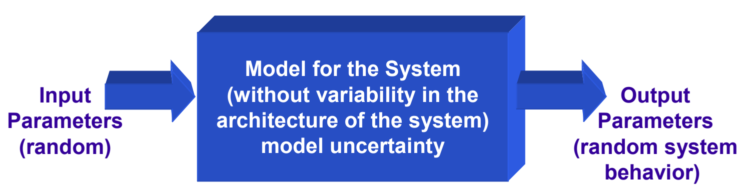


General Procedure

- The analytical and computational steps that are needed for performing Monte Carlo simulation are:
  1. Definition of the system
  2. Generation of random numbers
  3. Generation of random variables
  4. Evaluation of the model N times
  5. Statistical analysis of the resulting behavior
  6. Study of efficiency and convergence

System Definition

- The definition of the system should includes its boundaries, input parameters, output (or behavior) measures, architecture, and models that relate the input parameters and architecture to the output parameters.



Generation of Random Numbers

- Random numbers are real values, if normalized using the largest possible value, result in real values in the range [0,1].
- Random numbers have a uniform distribution on the range [0,1].
- A set of random numbers should also satisfy the condition of non-correlation for the purpose of simulation use.

Generation of Random Numbers

- Significance
  - Their transformation into real values that follow any distribution of interest. They constitute the basis for random variable generation
- Types
  - Mechanical
  - Tabulated
  - Computer based (recursive functions) using a seed

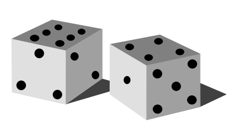
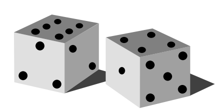


Table of Random Numbers in the Range [0,1]

0.538246	0.181648	0.172614	0.450166	0.293027	0.030195	0.757836	0.915061
0.663357	0.368934	0.516388	0.656254	0.284258	0.906335	0.329788	0.054487
0.035771	0.053784	0.424573	0.942479	0.293872	0.326815	0.862351	0.358055
0.51356	0.165508	0.667312	0.878444	0.414203	0.100839	0.555287	0.685601
0.880006	0.069305	0.85441	0.371911	0.751341	0.128446	0.678679	0.514995
0.880006	0.069305	0.85441	0.371911	0.751341	0.128446	0.678679	0.514995
0.748794	0.902497	0.629615	0.662531	0.932879	0.018376	0.683876	0.55481
0.115441	0.207278	0.887853	0.812124	0.082143	0.939258	0.666874	0.582525
0.953369	0.543997	0.806486	0.707493	0.503949	0.489926	0.774467	0.248617
0.2436	0.537111	0.181388	0.619277	0.131852	0.131876	0.361814	0.582682
0.610186	0.41158	0.339972	0.080869	0.429448	0.82277	0.63269	0.863227
0.848375	0.043973	0.071429	0.713405	0.56201	0.71605	0.53662	0.357681
0.102922	0.201752	0.61727	0.416471	0.371492	0.633301	0.857578	0.483474
0.009326	0.912932	0.11385	0.3316	0.852807	0.626191	0.035676	0.581386
0.801494	0.365068	0.54875	0.480788	0.032959	0.906331	0.291263	0.706212
0.682049	0.946008	0.960047	0.830463	0.186225	0.123762	0.674147	0.012839

Generation of Random Variables

- Inverse Transformation Method
  - A random number u is first generated in the range [0,1]
  - Then the value (x) of a generated continuous random variable, X, is determined as follows:

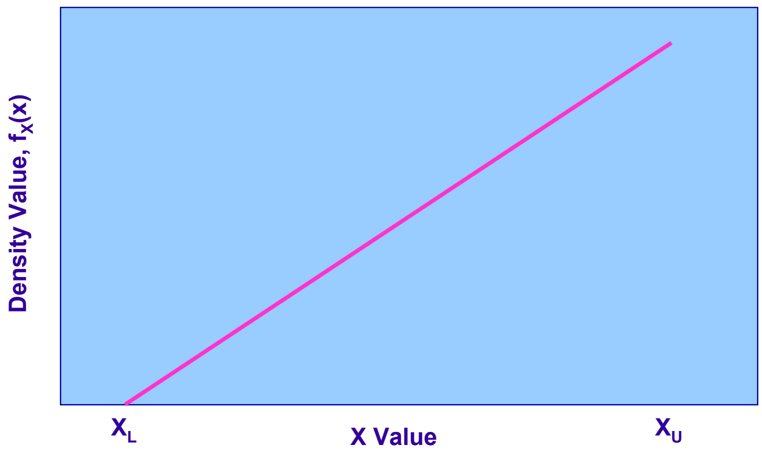


$x = F_X^{-1}(u)$ = **the inverse of the cumulative distribution function of the random variable X evaluated at u.**

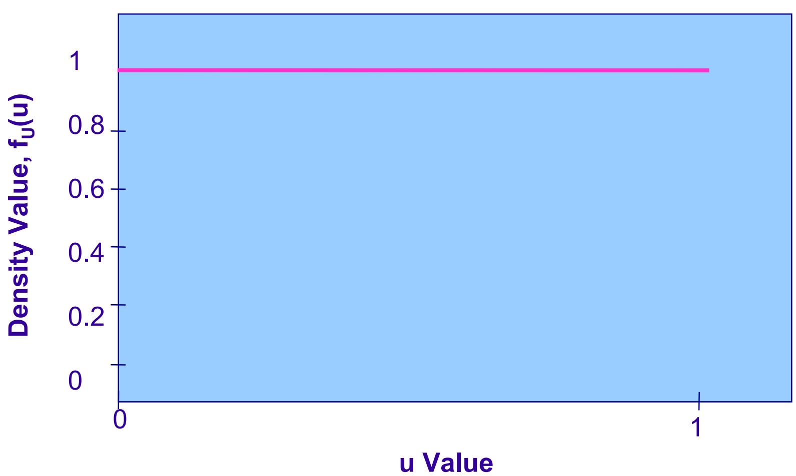
Since the range of FX(x) is in the range [0,1], a unique value for x is obtained all the time in each simulation cycle.

Triangular Distribution

- Triangular Density Function



Uniform Density Function



Cumulative Function

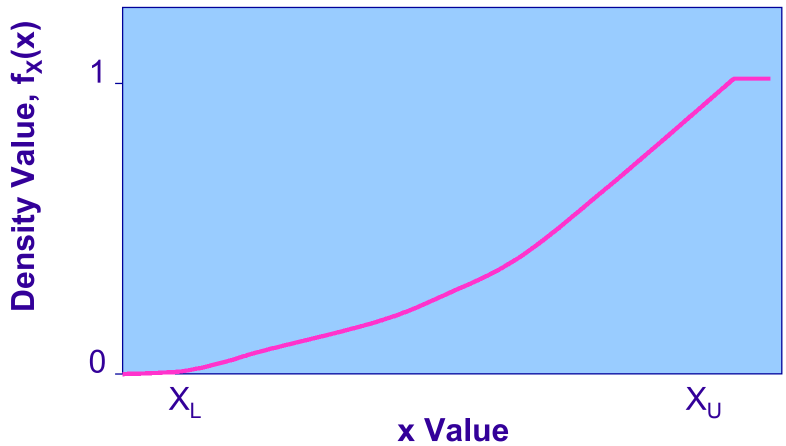


Illustration of inverse transformation

Normal Distribution

- For mean m and standard deviation s, the uniform variate is first transformed to the standard normal deviate z, which is then transformed to normal deviate x by:

$$x = \mu + z\sigma$$

Normal Distribution (cont'd)

- Consider the case where the sample consists of the following:

  U = {0.82, 0.43, 0.71, 0.18, 0.66}

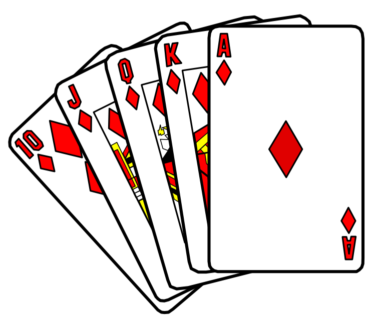
- To find the corresponding z values, enter the standard normal table for a probability of 0.82, for example, and read the z value (z = 0.915). Continuing for each ui value yields:

  Z = {0.915, -0.176, 0.556, -0.915, 0.413}

- Then use $x = \mu + z\,\sigma$ to obtain the values of X.

Other Methods for Generating Random Variables:

- Specialized
- Efficient
- Automated



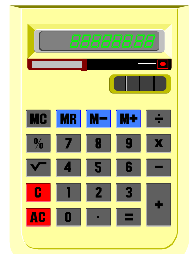
Evaluation of Model

Substitute the generated input random variables (X_i) into an analytical model, g($\underline{X}$) to predict a response Y. The model can be a simple education or a complex computer code.

$$Y = g(X_1, X_2, X_3, \ldots, X_n)$$

This process is repeated N times.

- Noncorrelated random variables
- Correlated random variables



Statistical Analysis of Results

■Mean response:

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{n} Y_i$$

■Variance:

$$Var(Y) = \frac{1}{N-1}\left[\sum_{i=1}^{N} Y_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} Y_i\right)^2\right]$$

■Variance of estimated mean:

$$Var(\bar{Y}) = \frac{1}{N}\frac{1}{N-1}\left[\sum^{N} Y_i^2 - \frac{1}{N}\left(\sum^{N} Y_i\right)^2\right]$$

Study of Efficiency and Convergence Study of

By increasing N, the variance of the estimated mean (Var($\bar{Y}$)) decreases. Three issues of interest:

- Bias
- Convergence
- Statistical accuracy

Methods to improve accuracy and efficiency.

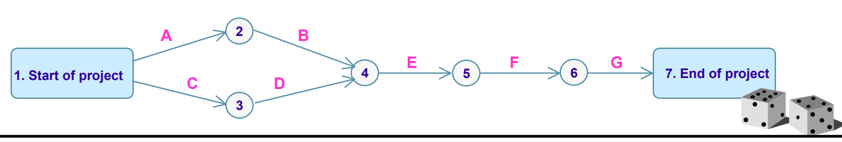
Variance reduction techniques:

- Importance sampling
- Conditional expectation
- Antithetic variates

Case Study

- Example 7.3 Warehouse Construction
  - A warehouse is to be constructed from precast elements with the following construction tasks:

A: excavation of foundations
B: construction of foundations
C: construction of precast elements at factory
D: transportation of precast elements to construction site
E: assembly of elements at site
F: construction of roof
G: exterior and interior finishing



Case Study

■ Random Variables:

Task	Name	Mean (days)	Standard Deviation (days)	Distribution Type
A	foundation excavation	3	1	Normal
B	foundation construction	2	0.5	Normal
C	precast-elements construction	4	1	Normal
D	transportation of elements	0.5	0.5	Normal
E	assembly of elements	4	1	Normal
F	roofing	2	1	Normal
G	finishing	3	1	Normal

Model:
The project completion time, T, is a random variable that is given by:

$$T = \max\{A+B\ ,\ C+D\} + E + F + G$$

Case Study

■ Random Numbers:

Task A	Task B	Task C	Task D	Task E	Task F	Task G
0.642707	0.758002	0.547225	0.510713	0.924981	0.44491	0.671304
0.240297	0.092418	0.84715	0.071252	0.98112	0.793358	0.780596
0.169051	0.446979	0.990008	0.079644	0.391058	0.793205	0.276989
0.457609	0.52127	0.606333	0.006137	0.47927	0.121284	0.34367
0.386325	0.395759	0.956544	0.432595	0.723067	0.448813	0.008538
0.313708	0.061922	0.343042	0.230356	0.538481	0.63629	0.211676
0.137571	0.078837	0.471558	0.383158	0.203166	0.500447	0.101354
0.296782	0.610994	0.785467	0.285467	0.282056	0.560465	0.539651
0.908314	0.124274	0.709123	0.508328	0.496352	0.886927	0.720611
0.763968	0.327695	0.506164	0.246872	0.743617	0.275227	0.218178
0.139498	0.935402	0.789508	0.966422	0.440431	0.682035	0.476614
0.220256	0.040641	0.347426	0.282962	0.178687	0.092735	0.96486
0.344963	0.100168	0.963482	0.569873	0.933351	0.64664	0.858627
0.095613	0.791418	0.726318	0.376506	0.872995	0.895403	0.962331
0.22554	0.262949	0.63276	0.550859	0.198235	0.077169	0.08673
0.239485	0.985236	0.212528	0.445724	0.66247	0.32561	0.025242
0.191603	0.108613	0.897544	0.990706	0.933851	0.557361	0.050711
0.94601	0.241317	0.187334	0.015071	0.228146	0.832563	0.816427
0.973859	0.343243	0.19794	0.177672	0.125638	0.099943	0.747989
0.484109	0.214928	0.020997	0.424466	0.893968	0.866459	0.706856

Case Study

- Resulting duration: (allowing negative values!)

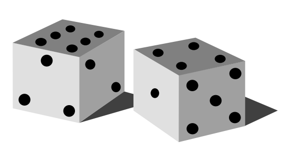
Task A	Task B	Task C	Task D	Task E	Task F	Task G	Completion Time
3.36526	2.3498042	4.1183849	0.5133909	5.4396627	1.8617675	3.443086	16.45958
2.2949296	1.3368915	5.0242746	-0.2334048	6.0778971	2.817944	3.7739874	17.4606982
2.0421427	1.933497	6.3270846	-0.2038625	3.7238635	2.8174073	2.4085497	15.0730432
2.8937885	2.0266006	4.2693511	-0.7523029	3.9481516	0.8313093	2.5979742	12.297824
2.7115217	1.8680395	5.7122992	0.4152843	4.5916169	1.8716354	0.6144956	13.205321
2.5150499	1.2304214	3.596267	0.1312882	4.0963777	2.3481148	2.1995787	12.389543
1.9086613	1.2934043	3.9288268	0.3516179	3.1698067	2.001117	1.7259547	11.177323
2.4667156	2.1407416	4.7905834	0.2118119	3.4240093	2.1518299	3.0993185	13.677553
4.3306524	1.4230146	4.5504403	0.5104081	3.9908831	3.2104754	3.5842925	16.539318
3.7188585	1.7770724	4.0154067	0.1579639	4.6542217	1.4032765	2.2218555	13.775285
1.9173874	2.7587914	4.8045186	1.4155181	3.8504386	2.472975	2.9415001	15.48495
2.2288943	1.1281585	3.6081638	0.2131524	3.079716	0.6756985	4.8105053	12.387236
2.6014871	1.3596149	5.793009	0.5878512	5.5015181	2.3758197	4.0742012	18.332399
1.6928452	2.4055804	4.601361	0.3428837	5.1407426	3.2559441	4.7787973	18.119729
2.2466222	1.6830249	4.3387303	0.5637742	3.1522166	0.5753632	1.6386032	10.268687
2.2923139	3.0884019	3.2025201	0.4319117	4.4187744	1.5483608	1.0437219	12.391573
2.128135	1.3829576	5.2678487	1.6770614	5.5053989	2.143973	1.3616434	15.955925
4.6076817	1.6491037	3.1123607	-0.5843253	3.2552784	2.9642836	3.9017196	16.378067
4.9412389	1.798406	3.1511551	0.0379101	2.8526594	0.7179441	3.6678706	13.978119
2.9602643	1.6053855	1.9659854	0.4049415	5.2480626	3.109864	3.5438339	16.46741

Case Study

Results:

- Mean value = 14.59 days
- Variance = 5.55 $(\text{days})^2$

Using different seeds (random numbers), the following can be obtained:



Case Study

- Resulting duration: (allowing negative values!)

Task A	Task B	Task C	Task D	Task E	Task F	Task G	Completion Time
3.2707799	1.34004	2.8529732	0.8156949	4.2147164	0.171328	3.5813727	12.578237
2.4095244	2.2375281	4.7058017	0.481743	3.4411537	2.5164544	1.7263661	12.871519
3.5991597	1.775744	2.6681462	0.7893469	4.9142851	3.3552784	4.3467318	17.991199
2.0844111	1.9638631	2.54382	0.2291748	4.1505155	1.8540637	3.6733168	13.72617
1.9739182	1.6473216	3.4610715	0.8680251	4.7929741	0.8324813	3.4212035	13.375756
2.0399005	1.0987845	4.0339016	1.0875602	4.6703954	0.8017872	4.6788003	15.272445
4.3768346	1.6527585	4.2793768	1.0147408	2.7347579	-0.3528514	3.8468091	12.258309
1.8453495	2.2365066	4.2789404	0.0851884	2.6055421	3.1097732	3.0369229	13.116367
2.593705	2.3210996	2.8829455	0.5873924	3.7176888	2.0464701	2.1752441	12.854208
5.1966454	2.1435484	5.366874	1.1363712	4.1090447	0.6695408	4.8949117	17.013691
2.5794094	2.1482732	3.7532931	0.1919199	5.3618234	0.7108243	3.113757	13.914087
3.2112634	1.96377	3.598772	0.2064942	2.8660028	1.6399625	2.5912269	12.272226
3.8344438	1.1952375	3.9198922	0.3809357	5.4304724	2.2215315	1.8570525	14.538738
3.4917621	1.0511701	1.9732268	-0.3812494	3.9582012	2.116783	2.3806975	12.998614
4.7463211	2.3356456	5.3719798	0.8047524	5.3859459	3.1270236	3.3903168	18.985253
2.5633476	2.1610288	4.5154829	0.8719888	3.7023022	2.5322679	1.4531975	13.075239
2.1647959	1.6361127	3.7908605	0.870752	2.1834205	2.5949514	2.7353984	12.175383
3.3407015	1.8485931	4.0957866	0.2755382	4.341604	1.0723773	2.6064022	13.209678
3.9968036	2.6271607	2.5352335	0.7837149	3.7496031	1.3063624	2.1972483	13.877178
3.0782144	1.9542634	5.674461	-0.2269675	3.6021733	2.1978306	2.2108603	13.45835

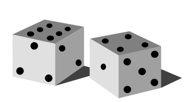
Case Study

Results (with new seed):

- **Mean value = 13.98 days**
- **Variance = 3.67 (days)^2**

Compare with:

- **Mean value = 14.59 days**
- **Variance = 5.55 (days)^2**



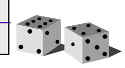
Problems

7-1. **The change in the length of a rod due to the axial force P is given by**

$$\Delta L = \frac{PL}{AE}$$

Where L = length of rod, P = applied axial force, A = cross sectional area of rod, and E = modulus of elasticity. Using 20 simulation cycles, determine the mean and variance of Δ L assuming noncorrelated random variables with the following probabilistic characteristics:

Random Variable	Mean Value	Coefficient of Variation	Distribution Type
P	100 kips	0.35	Lognormal
L	20 in	0.05	Normal
E	30,000 ksi	0.10	Lognormal
A	1 in^2	0.05	Normal



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1. **Ayyub, B.M., and McCuen, R., Probability, Statistics and Reliability for Engineers, CRC Press, FL, 1997.**
2. **Ang, and Tang, "Probability Concepts in Engineering and Planning Design," Volume II, John Wiley and Sons, NY, 1984.**
3. **Law, and Kelton, "Simulation Modeling and Analysis, "McGraw Hill, NY, 1982.**