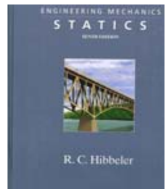
Engineering Mechanics: Statics Tenth Edition

CHAPTER 4d

FORCE SYSTEM RESULTANTS

UMBC

•College of Engineering •Department of Mechanical Engineering



by

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ENES 110 – Statics

Department of Mechanical Engineering

University of Maryland, Baltimore County

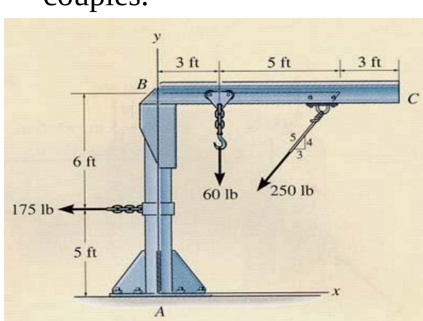
Equivalent Force-Couple Systems

Lecture's Objectives:

Students will be able to:

1) Determine the effect of moving a force.

2) Find an equivalent force-couple system for a system of forces and couples.



In-Class Activities:

- Reading Quiz
- Applications
- Equivalent Systems
- System reduction
- Concept quiz
- Group problem solving
- Attention quiz

Reading Quiz

1. A **general system** of forces and couple moments acting on a rigid body can be reduced to a ___ .

   A) single force.

   B) single moment.

   C) single force and two moments.

   D) single force and a single moment.

2. The original force and couple system and an equivalent force-couple system have the same _____ effect on a body.

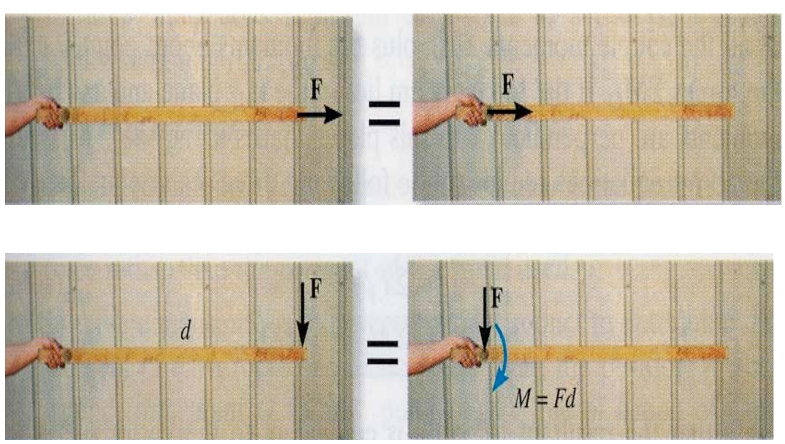
   A) internal

   B) external

   C) internal and external

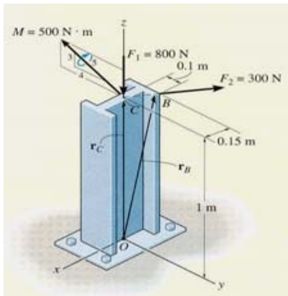
   D) microscopic

Applications



What is the resultant effect on the person's hand when the force is applied in four different ways ?

Applications (cont'd)



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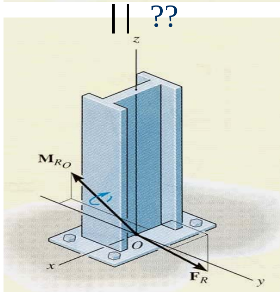
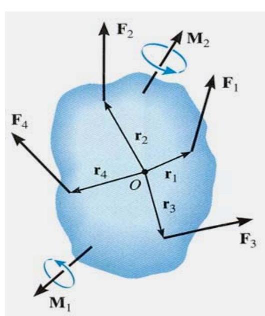


Several forces and a couple moment are acting on this vertical section of an I-beam.

Can you replace them with just one force and one couple moment at point *O* that will have the same external effect? If yes, how will you do that?

An Equivalent System (Section 4.7)



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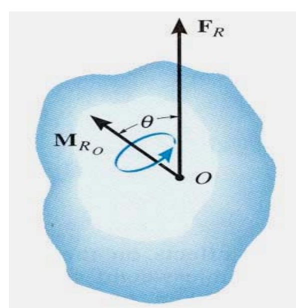
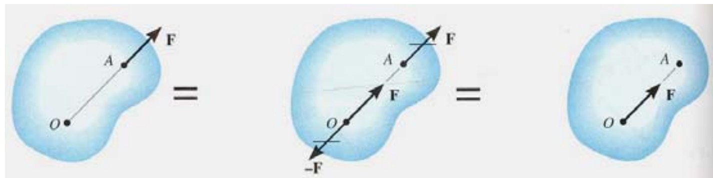


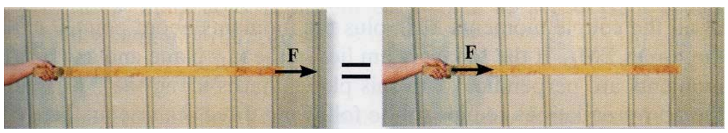
When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect

The two force and couple systems are called equivalent systems since they have the same external effect on the body.

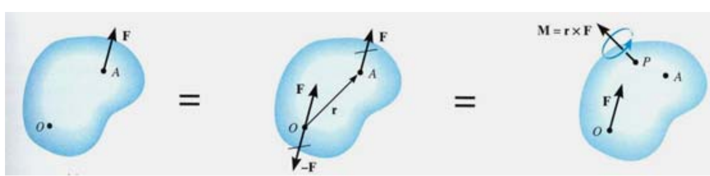
Moving a Force on its Line of Action



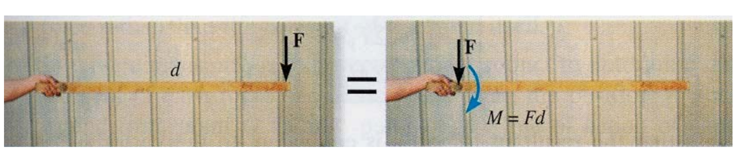
Moving a force from *A* to *O*, when both points are on the vector's line of action, does not change the external effect. Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).



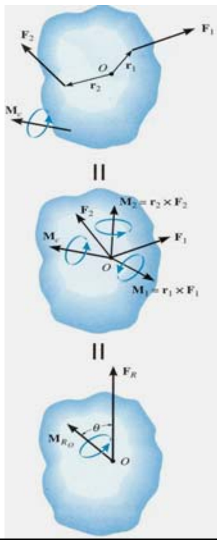
Moving a Force Off of its Line of Action



Moving a force from point *A* to *O* (as shown above) requires creating an additional couple moment. Since this new couple moment is a "free" vector, it can be applied at any point *P* on the body.



Finding the Resultant of a Force and Couple System (Section 4.8)

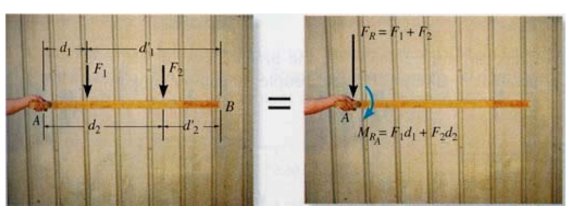


When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point *O*.

Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

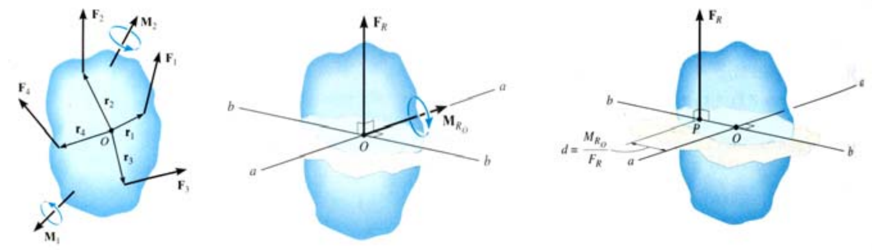
Resultant of a Force and Couple System



If the force system lies in the *x-y* plane (the 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{R_x} = \Sigma F_x$$
$$F_{R_y} = \Sigma F_y$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

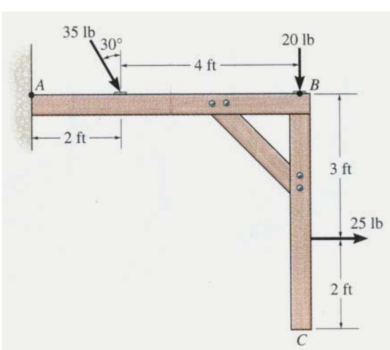
Reducing a Force-Moment to a Single Force (Section 4.9)



If $\mathbf{F}_R$ and $\mathbf{M}_{RO}$ are perpendicular to each other, then the system can be further reduced to a single force, $\mathbf{F}_R$, by simply moving $\mathbf{F}_R$ from *O* to *P*.

In three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force.

Example 1



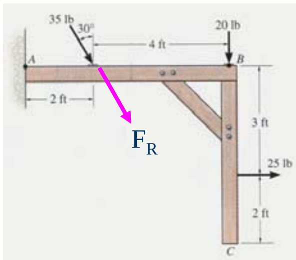
Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at *A* and then the equivalent single force location along the beam *AB*.

Plan:

1) Sum all the *x* and *y* components of the forces to find F_{RA}.
2) Find and sum all the moments resulting from moving each force to *A*.
3) Shift the F_{RA} to a distance *d* such that $d = M_{RA}/F_{Ry}$

Example 1 (cont'd)



$+\rightarrow \Sigma F_{Rx} = 25 + 35 \sin 30° = 42.5$ lb

$+\downarrow \Sigma F_{Ry} = 20 + 35 \cos 30° = 50.31$ lb

$+\circlearrowleft M_{RA} = 35 \cos 30° (2) + 20(6) - 25(3)$

$= 105.6$ lb·ft

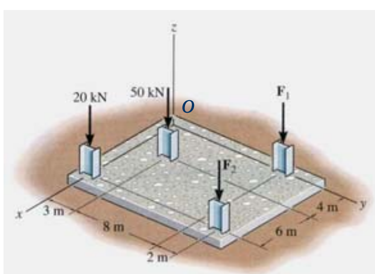
$F_R = (42.5^2 + 50.31^2)^{1/2} = 65.9$ lb

$\theta = \tan^{-1} (50.31/42.5) = 49.8°$

The equivalent single force F_R can be located on the beam *AB* at a distance *d* measured from *A*.

$d = M_{RA}/F_{Ry} = 105.6/50.31 = 2.10$ ft.

Example 2



Given: The building slab has four columns. F_1 and $F_2 = 0$.

Find: The equivalent resultant force and couple moment at the origin *O*. Also find the location (*x*,*y*) of the single equivalent resultant force.

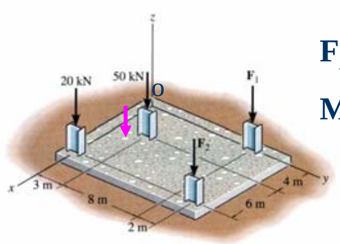
Plan:

1) Find $\mathbf{F}_{RO} = \sum \mathbf{F}_i = F_{Rzo}\,\mathbf{k}$

2) Find $\mathbf{M}_{RO} = \sum (\mathbf{r}_i \times \mathbf{F}_i) = M_{RxO}\,\mathbf{i} + M_{RyO}\,\mathbf{j}$

3) The location of the single equivalent resultant force is given as $x = -M_{RyO}/F_{RzO}$ and $y = M_{RxO}/F_{RzO}$

Example 2 (cont'd)



$\mathbf{F}_{RO} = \{-50\,\mathbf{k} - 20\,\mathbf{k}\} = \{-70\,\mathbf{k}\}$ kN

$\mathbf{M}_{RO} = (10\,\mathbf{i}) \times (-20\,\mathbf{k}) + (4\,\mathbf{i} + 3\,\mathbf{j}) \times (-50\,\mathbf{k})$

$= \{200\,\mathbf{j} + 200\,\mathbf{j} - 150\,\mathbf{i}\}$ kN·m

$= \{-150\,\mathbf{i} + 400\,\mathbf{j}\}$ kN·m

The location of the single equivalent resultant force is given as,

$x = -M_{Ryo}/F_{Rzo} = -400/(-70) = 5.71$ m

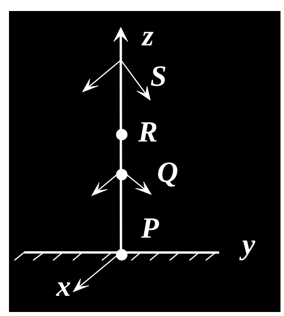
$y = M_{Rxo}/F_{Rzo} = (-150)/(-70) = 2.14$ m

Concept Quiz

1. The forces on the pole can be reduced to a single force and a single moment at point ____ .

   A) *P* B) *Q* C) *R*

   D) *S* E) Any of these points.



2. Consider two couples acting on a body. The simplest possible equivalent system at any arbitrary point on the body will have

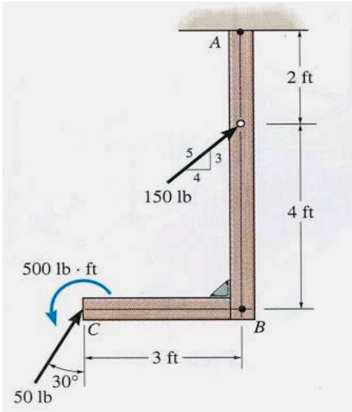
   A) one force and one couple moment.

   B) one force.

   C) one couple moment.

   D) two couple moments.

Example 3



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at *A*.

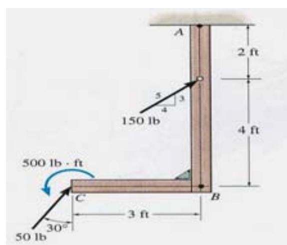
<u>Plan</u>:

1) Sum all the *x* and *y* components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to *A* and add them to the 500 lb - ft free moment to find the resultant M_{RA}.

Example 3 (cont'd)

Summing the force components:



$+ \rightarrow \Sigma F_x = (4/5)\,150 \text{ lb} + 50 \text{ lb} \sin 30° = 145 \text{ lb}$

$+ \uparrow \Sigma F_y = (3/5)\,150 \text{ lb} + 50 \text{ lb} \cos 30° = 133.3 \text{ lb}$

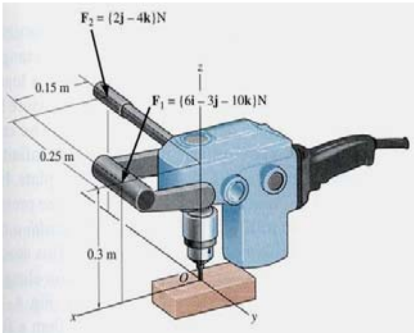
Now find the magnitude and direction of the resultant.

$F_{RA} = (145^2 + 133.3^2)^{1/2} = 197 \text{ lb}$ and $\theta = \tan^{-1}(133.3/145) = 42.6°$ ∠

$+\circlearrowleft M_{RA} = \{ (4/5)(150)(2) - 50 \cos 30° (3) + 50 \sin 30° (6) + 500 \}$

$= 760 \text{ lb}\cdot\text{ft}$

Example 4



Given: Handle forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied to the electric drill.

Find: An equivalent resultant force and couple moment at point *O*.

Plan:

a) Find $\mathbf{F}_{RO} = \Sigma\, \mathbf{F}_i$

b) Find $\mathbf{M}_{RO} = \Sigma\, \mathbf{M}_C + \Sigma\, (\, \mathbf{r}_i \times \mathbf{F}_i\,)$

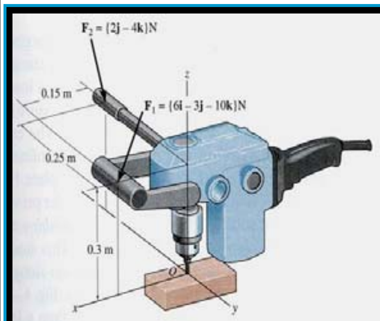
Where,

$\mathbf{F}_i$ = individual forces in Cartesian vector notation (CVN).

$\mathbf{M}_C$ = free couple moments in CVN (none in this example).

$\mathbf{r}_i$ = position vectors from the point *O* to any point on the line of action of $\mathbf{F}_i$.

Example 4 (cont'd)



$\mathbf{F}_1 = \{6\,\mathbf{i} - 3\,\mathbf{j} - 10\,\mathbf{k}\}$ N

$\mathbf{F}_2 = \{0\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k}\}$ N

$\mathbf{F}_{RO} = \{6\,\mathbf{i} - 1\,\mathbf{j} - 14\,\mathbf{k}\}$ N

$\mathbf{r}_1 = \{0.15\,\mathbf{i} + 0.3\,\mathbf{k}\}$ m

$\mathbf{r}_2 = \{-0.25\,\mathbf{j} + 0.3\,\mathbf{k}\}$ m

$\mathbf{M}_{RO} \quad \mathbf{r}_1 \times \mathbf{F}_1 \quad \mathbf{r}_2 \times \mathbf{F}_2$

$$\mathbf{M}_{RO} = \begin{vmatrix} i & j & k \\ 0.15 & 0 & 0.3 \\ 6 & -3 & -10 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & -0.25 & 0.3 \\ 0 & 2 & -4 \end{vmatrix}$$

$= \{0.9\,\mathbf{i} + 3.3\,\mathbf{j} - 0.45\,\mathbf{k} + 0.4\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}\}$ N·m

$= \{1.3\,\mathbf{i} + 3.3\,\mathbf{j} - 0.45\,\mathbf{k}\}$ N·m

Attention Quiz

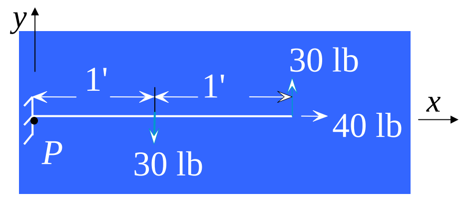
1. For this force system, the equivalent system at P is ____________ .

A) F_{RP} = 40 lb (along + x-dir.) and M_{RP} = +60 ft ·lb

B) F_{RP} = 0 lb and M_{RP} = +30 ft · lb

C) F_{RP} = 30 lb (along + y-dir.) and M_{RP} = -30 ft ·lb

D) F_{RP} = 40 lb (along + x-dir.) and M_{RP} = +30 ft ·lb



Attention Quiz (cont'd)

2. Consider three couples acting on a body. Equivalent systems will be _______ at different points on the body.

A) different when located

B) the same even when located

C) zero when located

D) None of the above.