CHAPTER

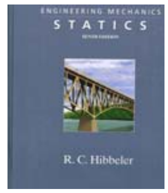
Engineering Mechanics: Statics Tenth Edition

FORCE VECTORS

UMBC

2b

•College of Engineering •Department of Mechanical Engineering



by
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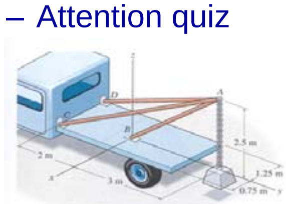
SPRING 2007

ENES 110 – Statics

Department of Mechanical Engineering
University of Maryland, Baltimore County

Lecture's Objectives

- Students will be able to :
  - a) Represent a 3-D vector in a Cartesian coordinate system.
  - b) Find the magnitude and coordinate angles of a 3-D vector.
  - c) Add vectors (forces) in 3-D space.
- In-Class Activities:
  - Reading quiz
  - Applications / Relevance
  - **A unit vector**
  - **3-D vector terms**
  - **Adding vectors**
  - Concept quiz
  - Examples
  - Attention quiz



READING QUIZ

1. Vector algebra, as we are going to use it, is based on a ___________ coordinate system.

A) Euclidean B) left-handed

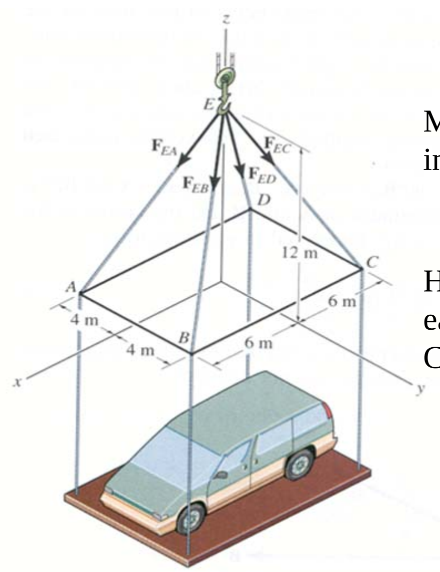
C) Greek D) right-handed E) Egyptian

2. The symbols α, β, and γ designate the __________ of a 3-D Cartesian vector.

A) unit vectors B) coordinate direction angles

C) Greek societies D) x, y and z components

APPLICATIONS

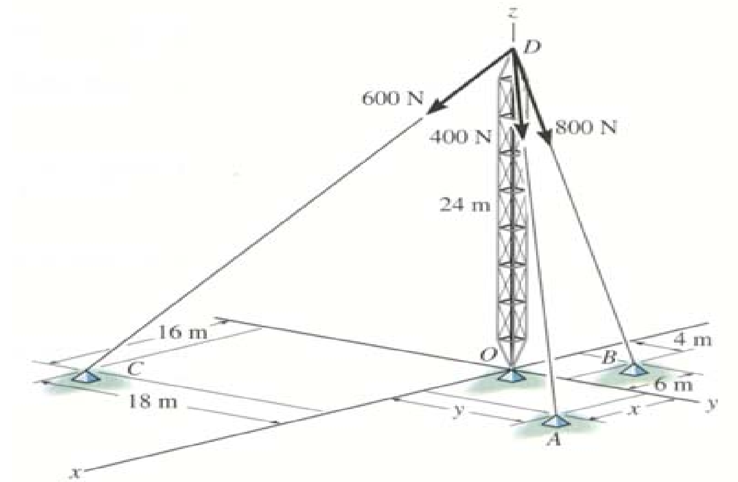


Many problems in real-life involve 3-Dimensional Space.

How will you represent each of the cable forces in Cartesian vector form?

APPLICATIONS (cont'd)

Given the forces in the cables, how will you determine the resultant force acting at *D*, the top of the tower?

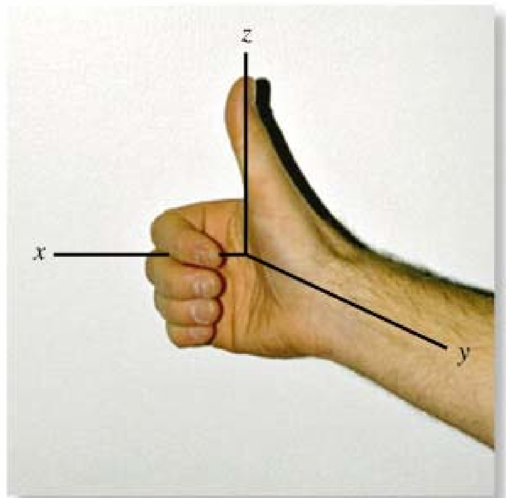


Cartesian Vectors

- A Cartesian coordinate system is often used to solve problems in three dimensions (3D).
- The coordinate system is *right-handed* which means that the thumb of the right hand points in the direction of the positive z-axis when the right hand fingers are curled about this axis and directed from the positive x toward the positive y-axis.

Cartesian Vectors

- Right-Handed Coordinate System

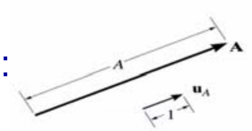


Cartesian Vectors

- Unit Vector
  - For a vector **A** with a magnitude of *A*, a unit vector is defined as

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} \qquad (2\text{-}3)$$

  - Characteristics of a unit vector:



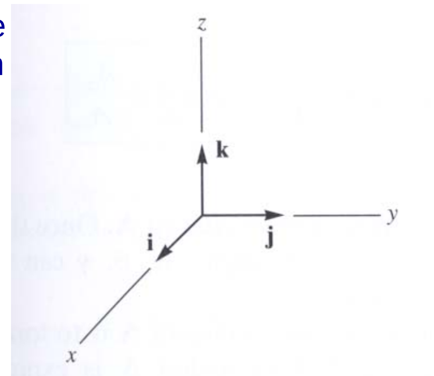
a) Its magnitude is 1.

b) It is dimensionless.

c) It points in the same direction as the original vector **A**.

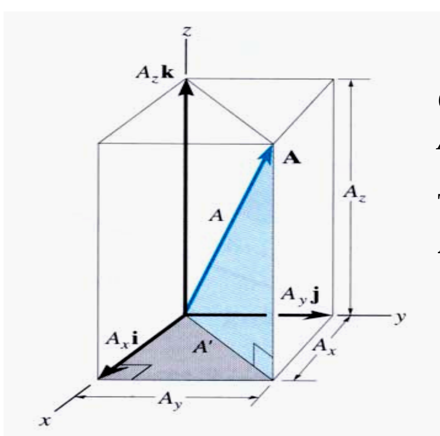
Cartesian Vectors

- Unit Vector (cont'd)
  - The unit vectors in the Cartesian axis system are **i**, **j**, and **k**. They are unit vectors along the positive *x*, *y*, and *z* axes respectively.



Cartesian Vectors

- 3-D Cartesian Vector Terminology



Consider a box with sides A_X, A_Y, and A_Z meters long.

The vector **A** can be defined as
A = (A_X **i** + A_Y **j** + A_Z **k**) m

Cartesian Vectors

- The projection of the vector **A** in the *x*-*y* plane is *A´*. The magnitude of this projection, *A´*, is found by using the same approach as a 2-D vector:

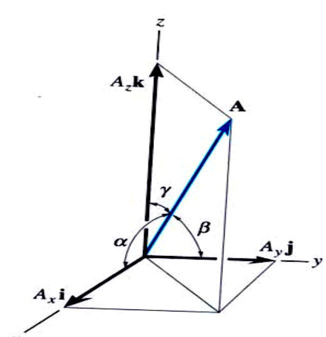
$$A' = |\mathbf{A}'| = \sqrt{A_x^2 + A_y^2}$$

- The magnitude of the position vector **A** can now be obtained as

$$A = |\mathbf{A}| = \sqrt{A'^2 + A_z^2} = \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad (2\text{-}6)$$

Cartesian Vectors

- 3-D Cartesian Vector Terminology (cont'd)
  – The direction or orientation of vector **A** is defined by the angles α, β, and γ.
  – Their range of values are from 0° to 180°.
  – Using trigonometry, "direction cosines" are found using the formulas



$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A} \qquad (2\text{-}7)$$

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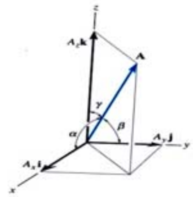
Cartesian Vectors

- 3-D Cartesian Vector Terminology (cont'd)



These angles are not independent. They must satisfy the following equation:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \qquad (2\text{-}10)$$

This result can be derived from the definition of a coordinate direction angles and the unit vector. Recall, the formula for finding the unit vector of any position vector:

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k} \qquad (2\text{-}8)$$

or written another way, $\mathbf{u}_A = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}$.

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Chapter 2b. FORCE VECTORS — Slide No. 13

Addition/Subtraction of Vectors

Once individual vectors are written in Cartesian form, it is easy to add or subtract them. The process is essentially the same as when 2-D vectors are added.

For example, if

$$\mathbf{A} = A_X\,\mathbf{i} + A_Y\,\mathbf{j} + A_Z\,\mathbf{k} \quad \text{and}$$

$$\mathbf{B} = B_X\,\mathbf{i} + B_Y\,\mathbf{j} + B_Z\,\mathbf{k}, \quad \text{then}$$

$$\mathbf{A} + \mathbf{B} = (A_X + B_X)\,\mathbf{i} + (A_Y + B_Y)\,\mathbf{j} + (A_Z + B_Z)\,\mathbf{k}$$

or

$$\mathbf{A} - \mathbf{B} = (A_X - B_X)\,\mathbf{i} + (A_Y - B_Y)\,\mathbf{j} + (A_Z - B_Z)\,\mathbf{k}\,.$$

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Addition/Subtraction of Vectors

- Important Notes

Sometimes 3-D vector information is given as:

a) Magnitude and the coordinate direction angles, or

b) Magnitude and projection angles.

You should be able to use both of these types of information to change the representation of the vector into the Cartesian form, i.e.,

$\boldsymbol{F} = \{10\,\mathbf{i} - 20\,\mathbf{j} + 30\,\mathbf{k}\}$ N .

CONCEPT QUESTIONS

1. If you know just $\mathbf{u}_A$, you can determine the ________ of **A** uniquely.

   A) magnitude

   B) angles (α, β and γ)

   C) components (A_X, A_Y, & A_Z)

   D) All of the above.

2. For an arbitrary force vector, the following parameters are randomly generated. Magnitude is 0.9 N, $\alpha = 30^\circ$, $\beta = 70^\circ$, $\gamma = 100^\circ$. What is wrong with this 3-D vector ?

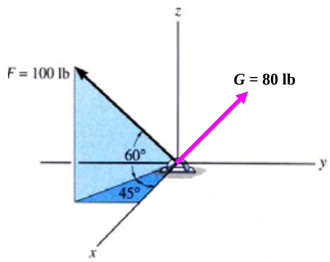
   A) Magnitude is too small.

   B) Angles are too large.

   C) All three angles are arbitrarily picked.

   D) All three angles are between 0° to 180°.

Example 1



Given: Two forces **F** and **G** are applied to a hook. Force **F** is shown in the figure and it makes 60° angle with the *x*-*y* plane. Force **G** is pointing up and has a magnitude of 80 lb with α = 111° and β = 69.3°.

Find: The resultant force in the Cartesian vector form.

Plan:

1) Using geometry and trigonometry, write **F** and **G** in the Cartesian vector form.
2) Then add the two forces.

Example 1 (cont'd)

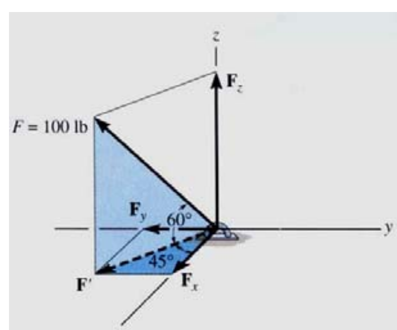
Solution : First, resolve force **F**.

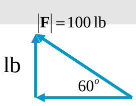
F_z = 100 sin 60° = 86.60 lb

F' = 100 cos 60° = 50.00 lb

F_x = 50 cos 45° = 35.36 lb

F_y = 50 sin 45° = 35.36 lb





$F_z = 86.6$ lb

$F' = 50$ lb

Now, you can write:

F = {35.36 **i** – 35.36 **j** + 86.60 **k**} lb

Example 1 (cont'd)

Now resolve force **G**.

We are given only α and β. Hence, first we need to find the value of γ.

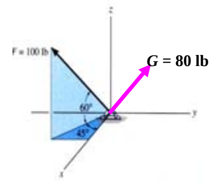
Recall the formula $\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$.

Now substitute what we know. We have

$\cos^2(111°) + \cos^2(69.3°) + \cos^2(\gamma) = 1$.

Solving, we get $\gamma = 30.22°$ or $120.2°$.

Since the vector is pointing up, $\gamma = 30.22°$



Now using the coordinate direction angles, we can get $\mathbf{u}_G$ and determine $\mathbf{G} = 80\ \mathbf{u}_G$ lb.

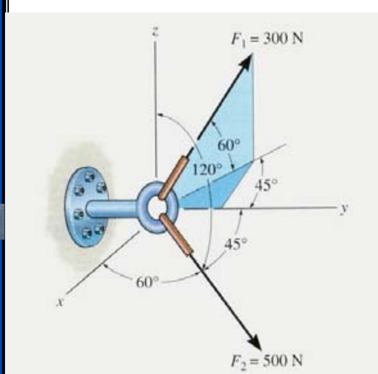
$\mathbf{G} = \{80\ [\cos(111°)\ \boldsymbol{i} + \cos(69.3°)\ \mathbf{j} + \cos(30.22°)\ \mathbf{k}\ ]\}$ lb

$\mathbf{G} = \{-28.67\ \mathbf{i} + 28.28\ \mathbf{j} + 69.13\ \mathbf{k}\ \}$ lb

Now, R = F + G or

$\mathbf{R} = \{6.69\ \mathbf{i} - 7.08\ \mathbf{j} + 156\ \mathbf{k}\}$ lb

Example 2



Given: The screw eye is subjected to two forces.

Find: The magnitude and the coordinate direction angles of the resultant force.

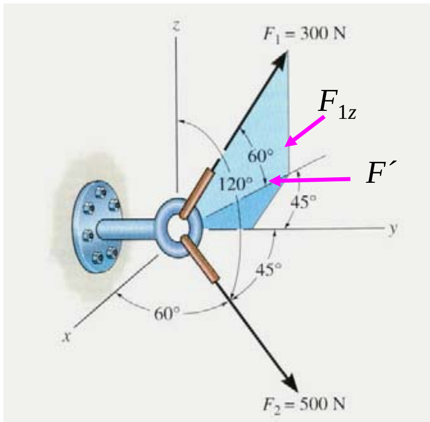
Plan:

1) Using the geometry and trigonometry, write $\mathbf{F}_1$ and $\mathbf{F}_2$ in the Cartesian vector form.

2) Add $\mathbf{F}_1$ and $\mathbf{F}_2$ to get $\mathbf{F}_R$.

3) Determine the magnitude and α, β, γ.

Example 2 (cont'd)



First resolve the force $\mathbf{F}_1$.

F_{1z} = 300 sin 60° = 259.8 N

F' = 300 cos 60° = 150.0 N

F' can be further resolved as,

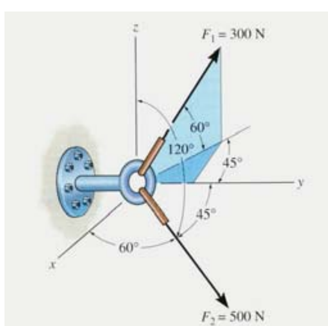
F_{1x} = -150 sin 45° = -106.1 N

F_{1y} = 150 cos 45° = 106.1 N

Now we can write :

$\mathbf{F}_1$ = {-106.1 **i** + 106.1 **j** + 259.8 **k** } N

Example 2 (cont'd)



The force $\mathbf{F}_2$ can be represented in the Cartesian vector form as:

$\mathbf{F}_2$ = 500{ cos 60° **i** + cos 45° **j** + cos 120° **k** } N

= { 250 **i** + 353.6 **j** – 250 **k** } N

$\mathbf{F_R} = \mathbf{F}_1 + \mathbf{F_2}$

= { 143.9 **i** + 459.6 **j** + 9.81 **k** } N

$F_R = \sqrt{143.9^2 + 459.6^2 + 9.81} = 481.7 = 482$ N

$\alpha = \cos^{-1}(F_{Rx} / F_R) = \cos^{-1}(143.9/481.7) = 72.6°$

$\beta = \cos^{-1}(F_{Ry} / F_R) = \cos^{-1}(459.6/481.7) = 17.4°$

$\gamma = \cos^{-1}(F_{Rz} / F_R) = \cos^{-1}(9.81/481.7) = 88.8°$

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ATTENTION QUIZ

1. What is not true about the unit vector, $\mathbf{u}_A$?

   A) It is dimensionless.

   B) Its magnitude is one.

   C) It always points in the direction of positive *x*- axis.

   D) It always points in the direction of vector **A**.

2. If **F** = {10 **i** + 10 **j** + 10 **k**} N and

   G = {20 **i** + 20 **j** + 20 **k** } N, then **F** + **G** = { ____ } **N**

   A) 10 **i** + 10 **j** + 10 **k**

   B) 30 **i** + 20 **j** + 30 **k**

   C) -10 **i** - 10 **j** - 10 **k**

   D) 30 **i** + 30 **j** + 30 **k**