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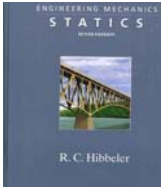
Engineering Mechanics: Statics Tenth Edition

CHAPTER

UMBC

2a

•College of Engineering •Department of Mechanical Engineering



by  
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**SPRING 2007**  
**ENES 110 – Statics**  
Department of Mechanical Engineering  
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UMBC Chapter 2a. FORCE VECTORS Slide No. 1

## Chapter Objectives

- To show how to add forces and resolve them into components using the Parallelogram Law.
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction.
- To introduce the dot product in order to determine the angle between two vectors or the projection of one vector onto another.

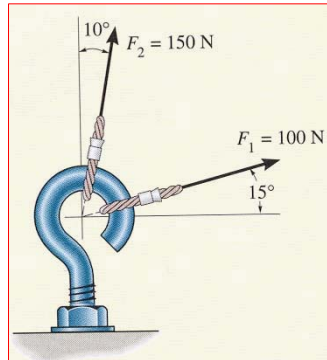
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## Two-Dimensional Vectors

### ■ Objectives

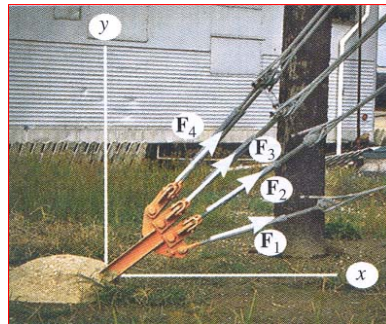
– Students will be able to:

- Resolve a 2-D vector into components
- Add 2-D vectors using Cartesian vector notations.



## Two-Dimensional Vectors

### ■ Application



There are four concurrent cable forces acting on the bracket.

How do you determine the resultant force acting on the bracket ?

## Scalar and Vectors

- Scalar
  - A quantity characterized by a positive or negative number.
- Vector
  - A quantity that has both a magnitude and direction.

## Scalar and Vectors

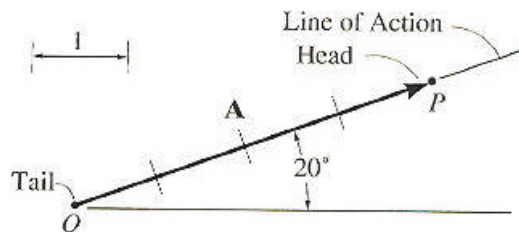
- Notations for Vectors
  - For handwritten work, a vector is generally represented by a letter with an arrow written over it, such as  $\vec{A}$
  - The magnitude is designated  $|\vec{A}|$  or  $A$ .
  - In your textbook vectors are symbolized in boldface type; for example,  $\mathbf{A}$  is used to designate the vector “A.” Its magnitude is symbolized by  $|\mathbf{A}|$ , or simply  $A$ .

## Scalar and Vectors

	<u>Scalars</u>	<u>Vectors</u>
Examples:	mass, volume	force, velocity
Characteristics:	It has a magnitude (positive or negative)	It has a magnitude and direction
Addition rule:	Simple arithmetic	Cartesian Components
Special Notation:	None	Bold font, a line, an arrow or a “carrot”

## Scalar and Vectors

### ■ Graphical Representation of a Vector



- The arrow represents the vector graphically and used to define the vector magnitude, direction, and sense.
- The magnitude is the length of the arrow, and the direction is defined by the angle between a reference axis and the arrow's line of action.
- The sense is indicated by the arrowhead.

# Vectors Operations

## ■ Multiplication or Division of a Vector by a Scalar

$$a\mathbf{A}$$

$$\text{magnitude: } |a\mathbf{A}| = |a||\mathbf{A}|$$

$$\frac{1}{a}\mathbf{A}$$

$$\text{magnitude: } \left| \frac{1}{a}\mathbf{A} \right| = \frac{1}{|a|}|\mathbf{A}|$$



Vector  $\mathbf{A}$  and its negative counterpart



Scalar Multiplication and Division

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# Vectors Operations

## ■ Vector Addition

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

- Two vectors  $\mathbf{A}$  and  $\mathbf{B}$  such as force or position, Fig. 2-4a, may be added to form a "resultant" vector  $\mathbf{R} = \mathbf{A} + \mathbf{B}$  by using the parallelogram law.

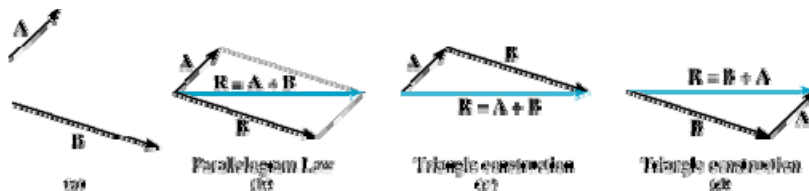
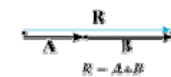


Fig. 2-4

Vector Addition



Addition of collinear vectors

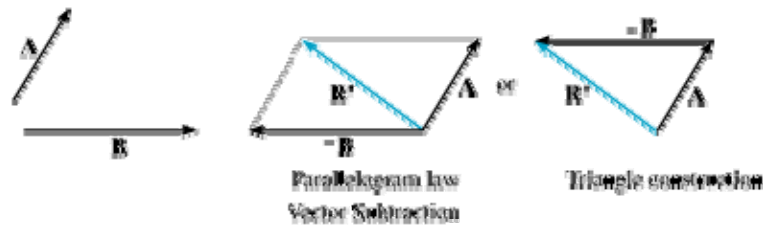
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# Vectors Operations

## ■ Vector Subtraction

- The resultant *difference* between two vectors **A** and **B** of the same type may be expressed as

$$\mathbf{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$



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# Vectors Operations

## ■ Resolution of a Vector

- A vector may be resolved into *components* having known lines of action by using the parallelogram law.

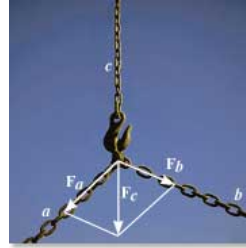
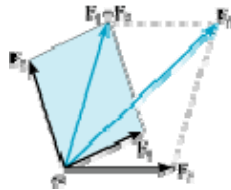


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## Vector Addition of Forces

### ■ Addition of Forces

Application: Chains



$$\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3.$$

## Vector Addition of Forces

### ■ Addition of Forces (cont'd)

- A force is vector quantity since it has a specified magnitude and direction.
- Two common problems in statics involve either finding the resultant force given its components or resolving a known force into components.
- The law of cosines is often used to find the magnitude, while the law of sines is used to find the direction.

## Vector Addition of Forces

- Law of Sines and Cosines



Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

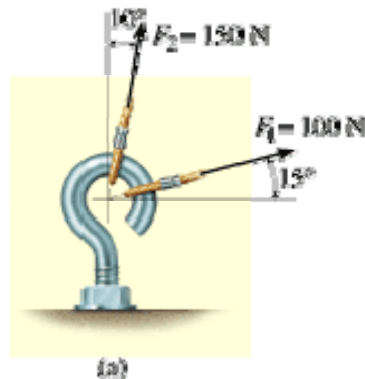
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

## Vector Addition of Forces

- Example 1

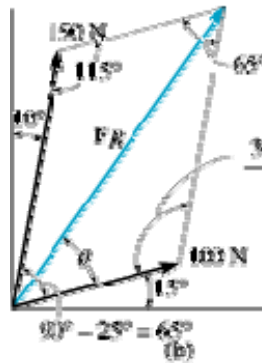
The screw eye in the figure is subjected to two forces  $F_1$  and  $F_2$ . Determine the magnitude and direction of the resultant force.



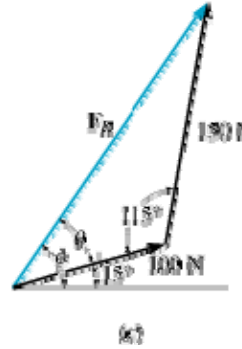


# Vector Addition of Forces

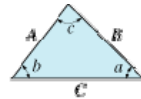
## ■ Example 1 (cont'd)



Parallelogram Law



Trigonometry

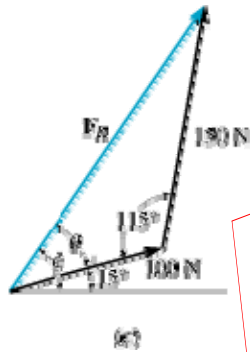


Sine law:  
 $\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$   
 Cosine law:  
 $C = \sqrt{A^2 + B^2 - 2AB \cos c}$

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# Vector Addition of Forces

## ■ Example 1 (cont'd)



Sine law:  
 $\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$   
 Cosine law:  
 $C = \sqrt{A^2 + B^2 - 2AB \cos c}$

$$F_r = \sqrt{(100)^2 + (150)^2 - 2(100)(150)\cos 115} = 212.6 = 213 \text{ N}$$

$$\frac{150}{\sin \theta} = \frac{212.6}{\sin 115^\circ} \Rightarrow \theta = \sin^{-1}\left(\frac{150 \sin 115}{212.6}\right) = 39.8^\circ$$

$$\phi = 39.8^\circ + 15^\circ = 54.8^\circ$$

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## Addition of a System of Coplanar Forces (2D)

- In the plane, a force can be resolved into two rectangular components.
- There are two separate notations for doing this:
  - Scalar Notation: we write the force  $\mathbf{F}$  as  $(F_x, F_y)$  where  $F_x$  and  $F_y$  are scalar components of the force  $\mathbf{F}$  in the directions of the positive  $x$ - and  $y$ -axes, respectively. If  $F_x$  and  $F_y$  are negative, it means that  $|F_x|$  and  $|F_y|$  are directed along the negative  $x$ - and  $y$ -axes.

## Addition of a System of Coplanar Forces (2D)

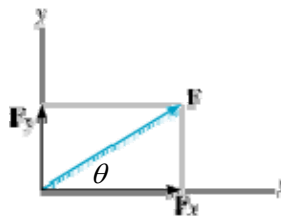
- Cartesian Vector Notation: we write the force  $\mathbf{F}$  as

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$$

- Where  $\mathbf{i}$  and  $\mathbf{j}$  represent the positive directions of the  $x$ - and  $y$ -axes, respectively.

$$F_x = F \cos \theta$$

$$F_y = F \sin \theta$$



## Addition of a System of Coplanar Forces (2D)

- The resultant of several coplanar forces can easily be determined if an  $x, y$ -coordinate system is established and the forces are resolved along the axis. For example,

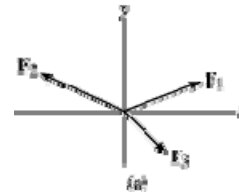
$$\mathbf{F}_1 = F_{1x}\mathbf{i} + F_{1y}\mathbf{j}$$

$$\mathbf{F}_2 = F_{2x}\mathbf{i} + F_{2y}\mathbf{j}$$

$$\mathbf{F}_3 = F_{3x}\mathbf{i} + F_{3y}\mathbf{j}$$

⋮

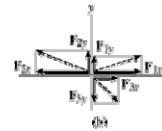
$$\mathbf{F}_n = F_{nx}\mathbf{i} + F_{ny}\mathbf{j}$$



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## Addition of a System of Coplanar Forces (2D)

Then the resultant is given by



$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \cdots + \mathbf{F}_n \\ &= (F_{1x} + F_{2x} + F_{3x} + \dots + F_{nx})\mathbf{i} + (F_{1y} + F_{2y} + F_{3y} + \dots + F_{ny})\mathbf{j} \\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j} \end{aligned}$$

- In general case, the  $x$  and  $y$  components of the resultant of any number of coplanar forces can be represented symbolically by the algebraic sum of the  $x$  and  $y$  components of all the forces, that is

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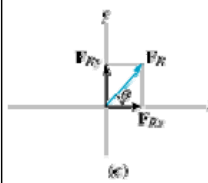
## Addition of a System of Coplanar Forces (2D)

$$F_{Rx} = \sum F_x$$

$$F_{Ry} = \sum F_y$$

*x* and *y* components of Resultant

- The magnitude and direction of the resultant force are given by



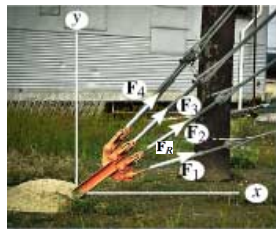
$$|\mathbf{F}_R| = F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right|$$

Magnitude & Direction

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## Addition of a System of Coplanar Forces (2D)



The resultant force of the four cable forces acting on the supporting bracket can be determined by adding algebraically the separate *x* and *y* components of each cable force.

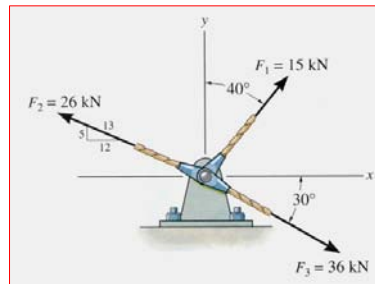
This resultant  $\mathbf{F}_R$  produces the *same pulling effect* on the bracket as all four cables.

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## Addition of a System of Coplanar Forces (2D)

### ■ Example 2

The Three concurrent forces are acting on a bracket. Find the magnitude and the angle of the resultant force.



## Addition of a System of Coplanar Forces (2D)

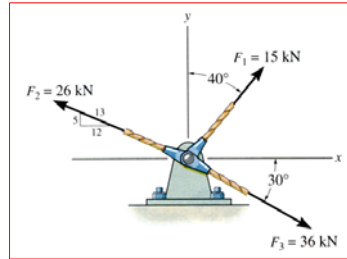
### ■ Example 2 (cont'd)

Plan:

- Resolve the forces in their  $x$ - $y$  components.
- Add the respective components to get the resultant vector.
- Find magnitude and angle from the resultant components.

# Addition of a System of Coplanar Forces (2D)

## ■ Example 2 (cont'd)



$$\mathbf{F}_1 = \{ 15 \sin 40^\circ \mathbf{i} + 15 \cos 40^\circ \mathbf{j} \} \text{ kN}$$

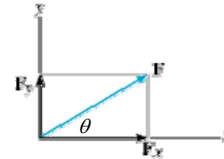
$$= \{ 9.642 \mathbf{i} + 11.49 \mathbf{j} \} \text{ kN}$$

$$\mathbf{F}_2 = \{ -(12/13)26 \mathbf{i} + (5/13)26 \mathbf{j} \} \text{ kN}$$

$$= \{ -24 \mathbf{i} + 10 \mathbf{j} \} \text{ kN}$$

$$\mathbf{F}_3 = \{ 36 \cos 30^\circ \mathbf{i} - 36 \sin 30^\circ \mathbf{j} \} \text{ kN}$$

$$= \{ 31.18 \mathbf{i} - 18 \mathbf{j} \} \text{ kN}$$



$$F_x = F \cos \theta$$

$$F_y = F \sin \theta$$

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# Addition of a System of Coplanar Forces (2D)

## ■ Example 2 (cont'd)

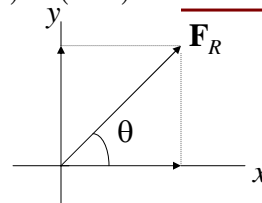
Summing up all the **i** and **j** components respectively, we get

$$\mathbf{F}_R = \{ (9.642 - 24 + 31.18) \mathbf{i} + (11.49 + 10 - 18) \mathbf{j} \} \text{ kN}$$

$$= \{ 16.82 \mathbf{i} + 3.49 \mathbf{j} \} \text{ kN}$$

$$|\mathbf{F}_R| = F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(16.82)^2 + (3.49)^2} = 17.2 \text{ kN}$$

$$\theta = \tan^{-1} \left| \frac{3.49}{16.82} \right| = 11.7^\circ$$



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# Addition of a System of Coplanar Forces (2D)

■ Example 3

To be discussed and solved in class

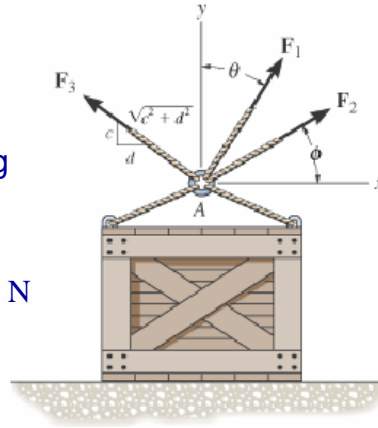
Determine the magnitude and direction measured counterclockwise from the positive  $x$  axis of the resultant force of the three forces acting on the ring A.

Given:

$$F_1 = 500 \text{ N}, F_2 = 400 \text{ N}, F_3 = 600 \text{ N}$$

$$\theta = 20^\circ, \phi = 30^\circ$$

$$c = 3, d = 4$$



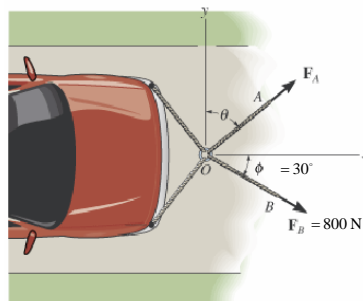
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# Addition of a System of Coplanar Forces (2D)

■ Example 4

To be discussed and solved in class

Determine the magnitude and direction  $\theta$  of  $F_A$  so that the resultant force is directed along the positive  $x$  axis and has a magnitude 1250 N.



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