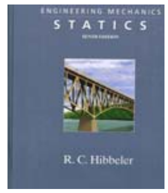
CHAPTER

Engineering Mechanics: Statics Tenth Edition

FORCE VECTORS

UMBC

•College of Engineering •Department of Mechanical Engineering



2a

by

Dr. Ibrahim A. Assakkaf

SPRING 2007

ENES 110 – Statics

Department of Mechanical Engineering

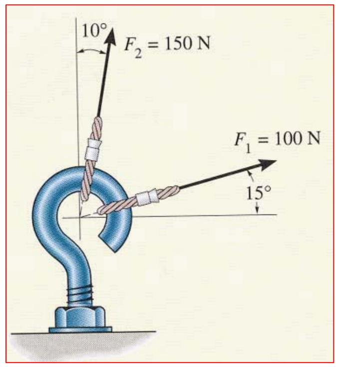
University of Maryland Baltimore County

Chapter Objectives

- To show how to add forces and resolve them into components using the Parallelogram Law.
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction.
- To introduce the dot product in order to determine the angle between two vectors or the projection of one vector onto another.

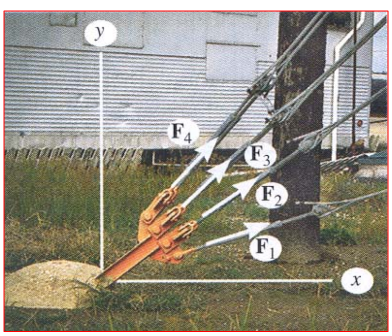
Two-Dimensional Vectors

- Objectives
  - Students will be able to:
    - a) Resolve a 2-D vector into components
    - b) Add 2-D vectors using Cartesian vector notations.



Two-Dimensional Vectors

- Application



There are four concurrent cable forces acting on the bracket.

How do you determine the resultant force acting on the bracket ?

Scalar and Vectors

- Scalar
  - A quantity characterized by a positive or negative number.
- Vector
  - A quantity that has both a magnitude and direction.

Scalar and Vectors

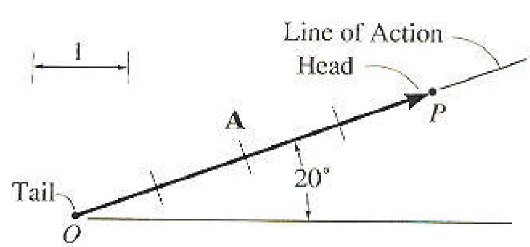
- Notations for Vectors
  - For handwritten work, a vector is generally represented by a letter with an arrow written over it, such as $\vec{A}$
  - The magnitude is designated $|\vec{A}|$ or A.
  - In your textbook vectors are symbolized in boldface type; for example, **A** is used to designate the vector "A." Its magnitude is symbolized by |*A*|, or simply *A*.

Scalar and Vectors

	Scalars	**Vectors**
Examples:	mass, volume	force, velocity
Characteristics:	It has a magnitude (positive or negative)	It has a magnitude and direction
Addition rule:	Simple arithmetic	Cartesian Components
Special Notation:	None	Bold font, a line, an arrow or a "carrot"

Scalar and Vectors

- Graphical Representation of a Vector



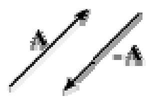
- The arrow represents the vector graphically and used to define the vector magnitude, direction, and sense.
- The magnitude is the length of the arrow, and the direction is defined by the angle between a reference axis and the arrow's line of action.
- The sense is indicated by the arrowhead.

Vectors Operations

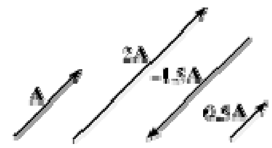
- Multiplication or Division of a Vector by a Scalar

$a\mathbf{A}$ $\qquad$ $\frac{1}{a}\mathbf{A}$

$\text{magnitude}: |a\mathbf{A}| = |a||\mathbf{A}|$ $\qquad$ $\text{magnitude}: \left|\frac{1}{a}\mathbf{A}\right| = \left|\frac{1}{a}\right||\mathbf{A}|$



Vector A and its negative counterpart



Scalar Multiplication and Division

Vectors Operations

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

- Vector Addition
  - Two vectors **A** and **B** such as force or position, Fig. 2–4a, may be added to form a "resultant" vector **R** = **A**+ **B** by using the parallelogram law.

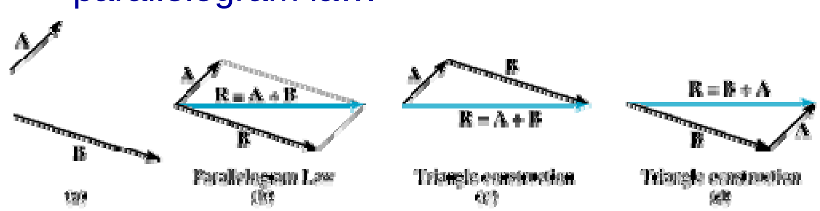
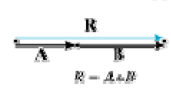


Fig. 2–4

Vector Addition



Addition of collinear vectors

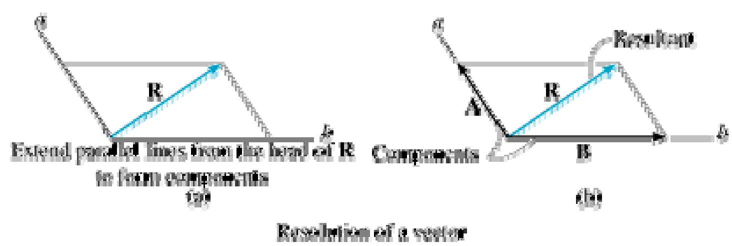
Vectors Operations

- Vector Subtraction
  - The resultant *difference* between two vectors **A** and **B** of the same type may be expressed as

$$\mathbf{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

Vectors Operations

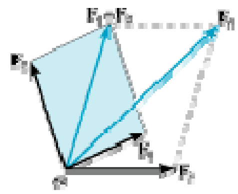
- Resolution of a Vector
  - A vector may be resolved into *components* having known lines of action by using the parallelogram law.



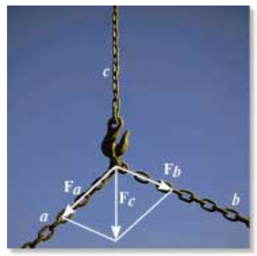
Vector Addition of Forces

- Addition of Forces

Application: Chains



$$\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3.$$

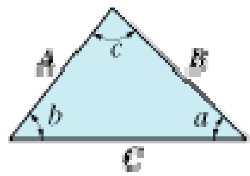


Vector Addition of Forces

- Addition of Forces (cont'd)
  - A force is vector quantity since it has a specified magnitude and direction.
  - Two common problems in statics involve either finding the resultant force given its components or resolving a known force into components.
  - The law of cosines is often used to find the magnitude, while the law of sines is used to find the direction.

Vector Addition of Forces

- Law of Sines and Cosines



Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

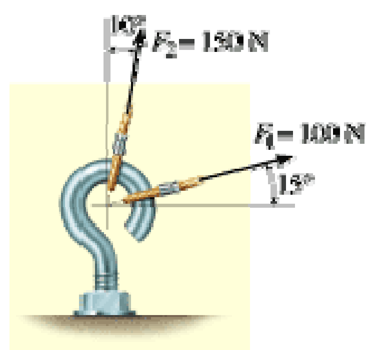
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Vector Addition of Forces

- Example 1

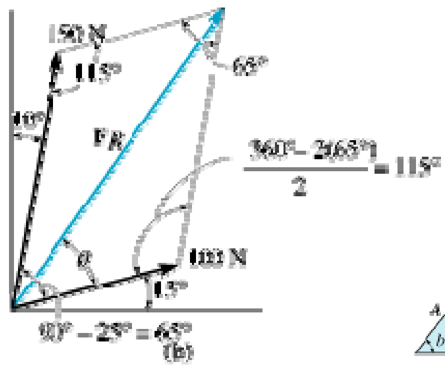
  The screw eye in the figure is subjected to two forces $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the magnitude and direction of the resultant force.



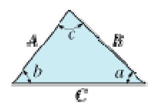
(a)

Vector Addition of Forces

- Example 1 (cont'd)



Parallelogram Law

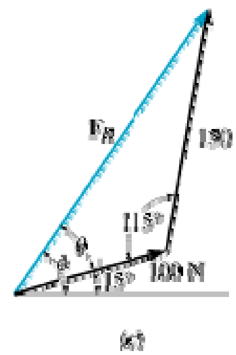


Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

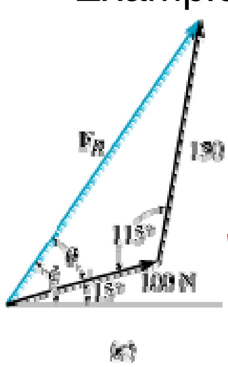
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

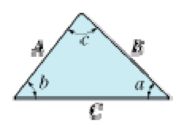


Trigonometry

Vector Addition of Forces

- Example 1 (cont'd)





Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

$$F_R = \sqrt{(100)^2 + (150)^2 - 2(100)(150)\cos 115} = 212.6 = \boxed{213 \text{ N}}$$

$$\frac{150}{\sin\theta} = \frac{212.6}{\sin 115^\circ} \Rightarrow \theta = \sin^{-1}\left(\frac{150 \sin 115}{212.6}\right) = 39.8^\circ$$

$$\phi = 39.8^\circ + 15^\circ = \boxed{54.8^\circ}$$

Addition of a System of Coplanar Forces (2D)

- In the plane, a force can be resolved into two rectangular components.
- There are two separate notations for doing this:
  – *Scalar Notation*: we write the force **F** as (F_x, F_y) where F_x and F_y are scalar components of the force **F** in the directions of the positive *x*- and *y*-axes, respectively. If F_x and F_y are negative, it means that $|F_x|$ and $|F_y|$ are directed along the negative *x*- and *y*-axes.

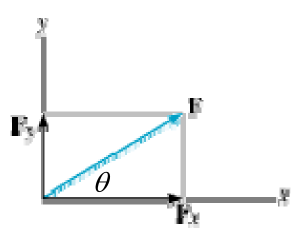
Addition of a System of Coplanar Forces (2D)

– *Cartesian Vector Notation*: we write the force **F** as

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

– Where **i** and **j** represent the positive directions of the *x*- and *y*-axes, respectively.

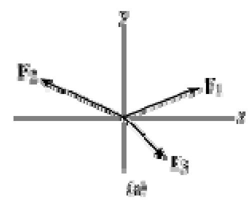
$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$



Addition of a System of Coplanar Forces (2D)

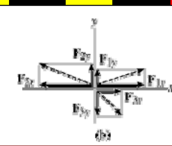
- The resultant of several coplanar forces can easily be determined if an x, y-coordinate system is established and the forces are resolved along the axis. For example,

$$
\begin{aligned}
\mathbf{F}_1 &= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} \\
\mathbf{F}_2 &= F_{2x}\mathbf{i} + F_{2y}\mathbf{j} \\
\mathbf{F}_3 &= F_{3x}\mathbf{i} + F_{3y}\mathbf{j} \\
&\vdots \\
\mathbf{F}_n &= F_{nx}\mathbf{i} + F_{ny}\mathbf{j}
\end{aligned}
$$



Addition of a System of Coplanar Forces (2D)

Then the resultant is given by



$$
\begin{aligned}
\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \cdots + \mathbf{F}_n \\
&= \left(F_{1x} + F_{2x} + F_{3x} + \ldots F_{nx}\right)\mathbf{i} + \left(F_{1y} + F_{2y} + F_{3y} + \ldots F_{ny}\right)\mathbf{j} \\
&= \left(F_{Rx}\right)\mathbf{i} + \left(F_{Ry}\right)\mathbf{j}
\end{aligned}
$$

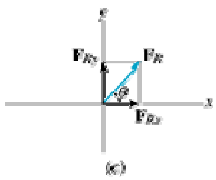
- In general case, the x and y components of the resultant of any number of coplanar forces can be represented symbolically by the algebraic sum of the x and y components of all the forces, that is

Addition of a System of Coplanar Forces (2D)

$$F_{Rx} = \sum F_x$$
$$F_{Ry} = \sum F_y$$

x and *y* components of Resultant

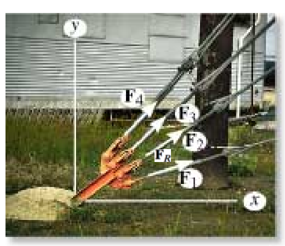
- The magnitude and direction of the resultant force are given by



$$\left|\mathbf{F}_R\right| = F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$
$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

Magnitude & Direction

Addition of a System of Coplanar Forces (2D)



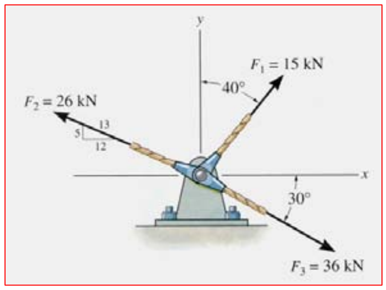
The resultant force of the four cable forces acting on the supporting bracket can be determined by adding algebraically the separate *x* and *y* components of each cable force.

This resultant $\mathbf{F}_R$ produces the *same pulling effect* on the bracket as all four cables.

Addition of a System of Coplanar Forces (2D)

- Example 2

  The Three concurrent forces are acting on a bracket. Find the magnitude and the angle of the resultant force.



Addition of a System of Coplanar Forces (2D)

- Example 2 (cont'd)

  Plan:

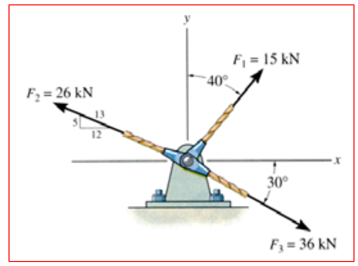
  a) Resolve the forces in their *x*-*y* components.

  b) Add the respective components to get the resultant vector.

  c) Find magnitude and angle from the resultant components.

Addition of a System of Coplanar Forces (2D)

- Example 2 (cont'd)



$\mathbf{F}_1$ = { 15 sin 40° **i** + 15 cos 40° **j** } kN

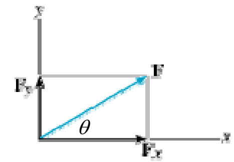
= { 9.642 **i** + 11.49 **j** } kN

$\mathbf{F}_2$ = { -(12/13)26 **i** + (5/13)26 **j** } kN

= { -24 **i** + 10 **j** } kN

$\mathbf{F}_3$ = { 36 cos 30° **i** – 36 sin 30° **j** } kN

= { 31.18 **i** – 18 **j** } kN



$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$

Addition of a System of Coplanar Forces (2D)

- Example 2 (cont'd)

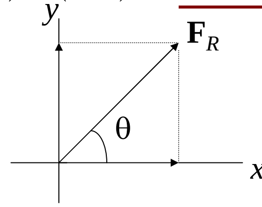
Summing up all the **i** and **j** components respectively, we get

$\mathbf{F}_R$ = { (9.642 – 24 + 31.18) **i** + (11.49 + 10 – 18) **j** } kN

= { 16.82 **i** + 3.49 **j** } kN

$$|\mathbf{F}_R| = F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(16.82)^2 + (3.49)^2} = \boxed{17.2\ \text{kN}}$$

$$\theta = \tan^{-1}\left|\frac{3.49}{16.82}\right| = \boxed{11.7^\circ}$$



Addition of a System of Coplanar Forces (2D)

To be discussed and solved in class

- Example 3

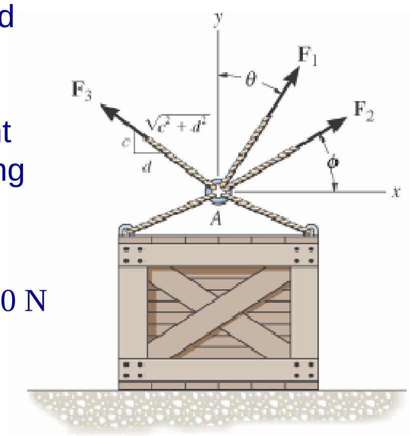
Determine the magnitude and direction measured counterclockwise from the positive x axis of the resultant force of the three forces acting on the ring *A.*

Given:

$F_1 = 500$ N, $F_2 = 400$ N, $F_3 = 600$ N

$\theta = 20^o$, $\phi = 30^o$

$c = 3$, $d = 4$



Addition of a System of Coplanar Forces (2D)

To be discussed and solved in class

- Example 4

Determine the magnitude and direction θ of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude 1250 N.